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Wittgenstein on Truth: Some Remarks on Paragraphs 134–137 of *Philosophical Investigations*

Keywords: Wittgenstein, truth, predicate, grammatical investigations

Słowa kluczowe: Wittgenstein, prawda, predykat, dociekania gramatyczne

Abstract

The main aim of this text is to explicate what paragraphs 134–137 of *Philosophical Investigations* say about truth. I start, however, with a discussion of the remarks on truth contained in the *Tractatus* because the thoughts expressed in the former should be read in the context of the thoughts expressed in the latter. According to my interpretation of paragraphs 134–137 of the *Investigations*, Wittgenstein aims to show that the relationship between the concept of truth and the concept of a proposition may consist in the fact that truth and falsity, in certain language games, are constitutive elements of what a proposition is, but it must be added that these concepts cannot be comprehended independently from each other. I also come to the conclusion that in his opinion the concept of truth is expressed in various ways in our language, it being conveyed by such expressions as “... is true” and “this is how things are: ...,” and that this shows that while asserting that something is true one can emphasize, on the one hand, that it is a *proposition that says* that things are a certain way, and on the other, just the fact that *things are thus and so*.

Introduction

Wittgenstein made relatively few direct remarks on the concept of truth, but this does not mean that the problem of truth is merely a peripheral issue in the context of his philosophy: On the contrary, it is related to the question of the nature of propositions, which lies at the heart of both his earlier and later approaches.

In the present text, I would like to limit myself to discussing what paragraphs 134–137 of *Philosophical Investigations* have to say about truth. Nevertheless, it is almost impossible to interpret these parts of the text properly without invoking thoughts expressed in Wittgenstein's earlier writings. This is because—as he himself points out in the *Preface* to the *Investigations*—the remarks contained there should be read in the context of those contained in the *Tractatus*:

Four years ago, however, I had occasion to reread my first book (the *Tractatus Logico-Philosophicus*) and to explain its ideas. Then it suddenly seemed to me that I should publish those old ideas and the new ones together: that the latter could be seen in the right light only by contrast with and against the background of my older way of thinking. (Wittgenstein, 2009, p. 4)

For this reason, I begin with a short presentation of Wittgenstein's approach to truth as contained in the *Tractatus*, before turning to a discussion of §§ 134–137 of the *Investigations*. My considerations regarding these paragraphs lead me to a number of conclusions. The first of these is that Wittgenstein formulates neither a theory, nor a definition of truth. The second is that he aims to show what relations hold between the concept of truth and the concept of a proposition. The third is that the relationship between these two concepts may consist, *inter alia*, in the fact that truth and falsity, in certain language games, are constitutive elements of what a proposition is, but it must be added that these concepts (*truth, proposition*) cannot be comprehended independently from each other. Finally, my fourth conclusion is that the concept of truth is expressed in various ways in our language, it being conveyed, *inter alia*, by such expressions as "... is true" and "this is how things are: ...," and that this shows that while asserting that something is true one can emphasize, on the one hand, that it is a *proposition that says* that things are a certain way, and on the other, just the fact that *things are*

thus and so. I end my text with considerations on three further issues: the question of how Wittgenstein approaches the problem of truth-bearers, the question of whether he ought to be considered an adherent of the redundancy conception of truth, and the question of what is new in the approach to truth contained in the *Investigations* compared to that contained in the *Tractatus*.

The *Tractatus* on Truth

In the *Tractatus*, Wittgenstein offers, *inter alia*, the following remarks on the subject of truth:

[...] The verb of the proposition is not “is true” or “is false”—as Frege thought—but that which “is true” must already contain the verb. (Wittgenstein, 1922, 4.063)

[...] One could e.g., believe that the words “true” and “false” signify two properties among other properties, and then it would appear as a remarkable fact that every proposition possesses one of these properties. This now by no means appears self-evident, no more so than the proposition “All roses are either yellow or red” would sound even if it were true. Indeed our proposition now gets quite the character of a proposition of natural science and this is a certain symptom of its being falsely understood. (Wittgenstein, 1922, 6.111)

A proposition is the expression of agreement and disagreement with the truth-possibilities of the elementary propositions. (Wittgenstein, 1922, 4.4)

Before I present the thoughts expressed in the quotes above that I take to be most essential for the present discussion, I wish to make an important terminological remark. The English word “proposition” in the translation of the *Tractatus* corresponds to the German word “*Satz*” as used in the book’s original version. Of course, this German word can be translated variously in different contexts, but it must be emphasized that, basically, the word “*Satz*” refers to a linguistic entity, not an abstract object such as is expressed *by* or *through* a sentence. So, this German word could be appropriately rendered in English by the term “sentence” in many contexts. Moreover, the elucidations in the *Tractatus* clearly show that Wittgenstein uses the word “*Sätze*”

to refer to meaningful sentences (cf. 3.11, 3.12, 3.141, 4.001).¹ Here, I wish to point out that these terminological findings also apply to the use of the word “*Satz*” in the *Investigations*: it is used in a way that denotes a meaningful sentence. So, the word “proposition” in the English translation of the *Investigations* should not be understood as referring to a certain abstract object that is expressed by some sentences. In this text, I also employ the word “proposition” to designate a meaningful sentence, and for this reason the words “proposition” and “sentence” are used interchangeably in many contexts below.

Wittgenstein’s statement in the *Tractatus* that the verb of the proposition is not “is true,” and that the verb must already be contained in that which is true, says that the concept of truth applies only to such things as already contain predicates. It may also be interpreted as suggesting that, in fact, the expression “is true” does not play the role of a predicate that is predicated of propositions. In my opinion, both of these points lead to the conclusion that, according to Wittgenstein, the logical form of such propositions as “John’s statement is true” should not be represented by the scheme “S is P” (“P(s)”). It is also compatible with Wittgenstein’s conviction that propositions do not *name* states of affairs, but *describe* them (cf. Wittgenstein, 1922, 3.1432, 3.144; Diamond, 2002).

The main idea expressed in the second quotation above is negative in character: Wittgenstein wishes to deny that truth and falsity are just certain properties among other properties. *That* approach to truth and falsity—the one he is criticizing—treats the statement that every proposition is either true or false as if it were itself a statement belonging to the natural sciences, and this in turn shows that the approach must be confused.

Proposition 4.4 says that every proposition “is the expression of agreement and disagreement with the truth-possibilities of the elementary propositions” (Wittgenstein 1922). This, in turn, implies that every proposition can be represented by a truth-table, and truth-tables—as is commonly known—are composed of, *inter alia*, the signs “T” and “F,” which have the same meaning as the words “true” and “false” (cf. Diamond 2003). So, the concepts of truth and falsity are constitutive for the concept of a proposition:

¹ Note that Diamond and Horwich—philosophers who in other respects construe many aspects of Wittgenstein’s thought so very differently—agree on this point (Diamond, 2002, 2003; Horwich, 2016, 2018).

One cannot comprehend what a proposition is if one does not grasp the concepts of truth and falsity. At the same time, it seems that the converse also holds: the concepts of truth and falsity cannot be grasped independently of the concept of a proposition. According to Wittgenstein, the signs which express these concepts do not stand for any objects:

It is clear that to the complex of the signs F and T no object (or complex of objects) corresponds; any more than to horizontal and vertical lines or to brackets. There are no logical objects. (Wittgenstein, 1922, 4.441)

This remark suggests that the concept of a proposition cannot be *defined* in terms of the concepts of truth and falsity, although grasping the latter is essential if we are to grasp the former. If truth and falsity were objects of any kind, they could be—in some sense—independent of any proposition, and for that reason their names could be used in a definition of the concept of a proposition; but, of course, they are not objects, and the concept of a proposition cannot be defined in that way.²

² Many commentators have interpreted the remarks on truth contained in the *Tractatus* as expressing a correspondence conception of truth (Black, 1964, p. 90; Hacker, 1981, p. 100; Hacker, 1986, p. 119; Stenius, 1981, p. 117). It should be added, however, that Hacker later changed his point of view on this issue: “But *if* we construe correspondence theories of truth as holding that being true is a relational property of sentences or propositions, then, despite these affinities, the *Tractatus* does not propound a correspondence theory of truth” (Baker & Hacker, 2005, p. 352). According to Glock, in the *Tractatus* Wittgenstein adopts a position similar in some respects to the correspondence conception of truth. This position is called the obtainment theory of truth and is characterized as follows: “The obtainment theory can be seen as a synthesis of correspondence, semantic and deflationary theories. It does justice to the idea that whether a sentence is true depends solely on what is the case. And it combines a ... semantic explanation of the relation between a sentence and what it ... says with a ... deflationary account of the agreement between what the sentence says and what ... is the case if it is true” (Glock, 2006, p. 347). However, in my opinion, the above-considered propositions from the *Tractatus* and the following remark from the *Notebooks* “‘p’ is true, says nothing else but p” (Wittgenstein, 1979, p. 9) should not be interpreted as an expression of a certain theory of truth, because as Wittgenstein himself points out in the *Tractatus*, “Philosophy is not a theory but an activity. ... The result of philosophy is not a number of philosophical propositions” (1922, 4.112).

How Should We Read the Sentence “‘p’ is true = p”?

Now I shall turn to §§ 134–137 of *Philosophical Investigations*. I would like to begin my discussion of Wittgenstein’s remarks on truth contained therein by pointing out that the main topic of this part of the *Investigations* is the question of the general form of the proposition. In these paragraphs, Wittgenstein is mainly concerned with criticism of his earlier (Tractarian) views on the general form of the proposition. So, it would be a mistake to treat his remarks on truth contained in §§ 134–137 of the *Investigations* as an attempt to explain this concept in terms of the concept of a proposition. Thus, commentators who claim—*contra* Horwich’s suggestions (cf. Horwich, 2012, p. 110)—that in these paragraphs Wittgenstein is not aiming to explain the concept of truth in terms of the previously defined concept of a proposition are right (cf. Bronzo, 2019, McFarland, 2020, Vision, 2005). Of course, that does not mean that paragraphs 134–137 are not written with the intention of elucidating the concept of truth in some way, and that this aim is not achieved at all in these paragraphs.

Having explained what the main topic of §§ 134–137 is, I now wish to turn to the most frequently discussed part of the text of these paragraphs:

‘p’ is true = p
 ‘p’ is false = not-p. (Wittgenstein 2009: § 136)

The remark quoted here is a commentary on the statement that the sentence “a proposition is whatever can be true or false” (Wittgenstein, 2009, § 136) is equivalent to the following *Tractarian* formulation of the general form of the proposition (Wittgenstein, 1922, 4.5): “Such and such is the case” (“Es verhält sich so und so”).³ Wittgenstein formulates this remark with the aim of elucidating the meaning of the expressions “is true” and “is false,” but, of course, does not assume that the concept of a proposition is independent of the concepts of truth and falsity. In my view, to properly understand the meaning of this remark two issues need to be addressed: the

³ Both in the *Tractatus* and in the *Investigations*, Wittgenstein uses the phrase “Es verhält sich so und so” which is, however, differently translated into English in the *Tractatus* and in the *Investigations*. In the first translation of the former, it is rendered as “Such and such is the case,” whereas in both translations of the latter it is rendered as “This is how things are.”

role played by the sign “=” in this part of the *Investigations*, and the question of how expressions corresponding to sentences enclosed in quotation marks should be understood.

It might seem that, in the scheme presented above, “=” is actually playing the role of a sign for equivalence (cf. Vision, 2005). Given that “‘p’ is true” is the scheme for a proposition, and “p” is a propositional variable, it seems natural to construe the sign which joins the former with the latter this way. However, in my opinion, this natural and, indeed, almost irresistible interpretation is mistaken. Firstly, it should be noted that, in his late writings, Wittgenstein uses not only a certain sign for identity, but also a certain sign for equivalence: for example, in *Remarks on the Foundations of Mathematics* (1991, pp. 178, 396). This puts into question the thesis that it is obvious that, in § 136, he is using the sign “=” to express equivalence. Secondly, in *Philosophical Grammar*, Wittgenstein calls the formula “ $\sim\sim p = p$ ” a rule of grammar (1978, p. 89), whereas in *Remarks on the Foundations of Mathematics* he calls a similar formula a proposition—this being the formula “ $\sim\sim p \equiv p$,” which differs from the former in that it contains a sign for equivalence instead of one for identity (1991, p. 178). This shows, in my opinion, that Wittgenstein’s use of a sign for identity between expressions that are schemas of propositions, not names, is not accidental. Thirdly, the following part of the text of *Remarks on the Foundations of Mathematics* shows that the schema under consideration does not so much express some true equivalence as elucidate the meaning of the expression “is true”:

For what does a proposition’s ‘being true’ mean? ‘p’ is true = p.
(That is the answer) (Wittgenstein, 1991, p. 117 (Appendix III, § 6))

Based on these three arguments, the following interpretative hypothesis can be formulated: The scheme “‘p’ is true = p” is a grammatical remark which expresses a grammatical rule, and the use of the sign “=” shows that instead of asserting that “p” is true one can simply assert that p, because both statements assert the same thing (cf. Wittgenstein, 2001, p. 106).⁴

⁴ This part of the text of Wittgenstein’s *Lectures* seems to confirm this interpretation; however, one must emphasize that according to the transcript of the *Lectures*, the expression “is true” is concatenated just with the propositional variable itself, not with

Let us now turn to our second question, namely, that of how expressions corresponding to sentences enclosed in quotation marks should be understood (this issue is considered by Baker & Hacker, 2005, Bartunek, 2019, and Vision, 2005). Of course, one cannot consider this one outside of the context in which it can be posed. For example, in case of the proposition “The sentence ‘*Paris is bigger than Rome*’ is italicized,” it is clear that enclosing the sentence in quotation marks serves to refer to a particular token of the sentential type “Paris is bigger than Rome,” and that this particular token is being treated merely as an inscription of a certain shape, whereas in the case of the proposition “The sentence ‘You are my Sun’ is not an especially original metaphor,” this serves to represent a certain meaningful English sentence, and not to refer to a certain inscription. So, one has to consider what role quotation marks play in the scheme “p” is true = p. Indeed, Wittgenstein addresses this issue explicitly in *Philosophical Grammar*:

So is it correct to write “‘p’ is true,” “‘p’ is false”; mustn’t it be “p is true” (or false)? The ink mark is after all not *true*; in the way in which it’s black and curved.

Does “‘p’ is true” state anything about the sign “p” then?

...

It can also be put thus: The proposition “‘p’ is true” can only be understood if one understands the grammar of the sign “p” as a propositional sign; not if “p” is simply the name of the shape of a particular ink mark. In the end one can say that the quotation marks in the sentence “‘p’ is true” are simply superfluous. (Wittgenstein, 1978, pp. 123–124)

He is claiming that the quotation marks in sentences of the form “‘p’ is true” do not serve to form the name of a sentence understood merely as an inscription. In the sentence “‘Paris is bigger than Rome’ is true,” the expression “‘Paris is bigger than Rome’” does not just refer to an inscription; rather, it represents a meaningful sentence. Moreover, Wittgenstein adds that the quotation marks in sentences of this type are, in the end, dispensable. This last remark does not appear in the *Investigations*. Why not? I think that the main reason is that such expressions as “Paris is bigger than Rome is true” are not—as Baker and Hacker (2005, p. 347) emphasize—grammatically correct. In the *Investigations*, Wittgenstein (2009, § 195, § 197)

the propositional variable enclosed in quotation marks. So, the scheme would have the following form: p is true = p.

recognizes that our ordinary ways of speaking are in perfect order, even if they sometimes suggest to us misleading interpretations of them. According to him, it is basically not the task of philosophy to *reform* language—that is, to introduce such a way of speaking, such a notation, as would be less vulnerable to misinterpretation, but rather to understand how we in fact use our language. Thus, in the case under consideration here, the point is not to substitute the sentence “‘Paris is bigger than Rome’ is true” with a certain artificial-sounding expression—for example, with the expression “Paris is bigger than Rome is true,” which might not then suggest that when one asserts that it is true that something is the case one is ascribing the property of being true to some object. The point is to see that the use of our original formulation—namely, “‘Paris is bigger than Rome’ is true”—differs *in principle*⁵ from the standard use of sentences of the form “S is P,” because the expression “‘Paris is bigger than Rome’,” which is a constituent of this expression, does not function in this case as the name of some object, and the expression “is true” does not in fact refer to some property or other. The fact that this is so can be shown by invoking the grammatical rule contained in paragraph 136: i.e., the schema “‘p’ is true = p.” If by means of a proposition of the form “‘p’ is true” one states the same as one does by means of a proposition of the form “p,” then the use of the former does not presuppose that in such a case the sentence enclosed in quotation marks is referring to some object, or that the property of being true is being ascribed to this object. However, in order to better understand what the difference between the use of sentences of the form “‘p’ is true” and a standard use of sentences of the form “S is P” consists in, we need to discuss the remarks subsequently contained in §§ 136–137 of the *Investigations*.

⁵ It is worth adding this qualification (in italics), because in some contexts the sentence “‘Paris is bigger than Rome’ is true” can be interpreted as saying that the sentence “Paris is bigger than Rome” expresses a truth in English. If one interprets the former sentence this way, then one can recognize it as having the form “S is P.” However, it should be added that in that case the assertion of identity “‘Paris is bigger than Rome’ is true = Paris is bigger than Rome” would not hold, as the fact that the sentence “Paris is bigger than Rome” expresses a truth in English amounts to something more than just this: that Paris is bigger than Rome.

How Does the Concept of Truth Relate to the Concept of a Proposition?

In § 136 of the *Investigations*, Wittgenstein contrasts two ways of relating concepts to each other. On the one hand, a given concept can fit another one, and on the other hand, it can belong to another one. According to the author of the *Investigations*, the use of the expressions “is true” and “is false” may be constitutive for some language-games in which we form propositions—that is, the use of these expressions belongs to the concept of a proposition expressed in these games, rather than merely fitting it:

And what a proposition is, is in *one* sense determined by the rules of sentence formation (in English, for example), and in another sense by the use of the sign in the language-game. And the use of the words “true” and “false” may also be a constituent part of this game; and we treat it as *belonging* to our concept ‘proposition’, but it doesn’t *fit* it. (Wittgenstein, 2009, § 136)

How, then, should we understand the two pictures to which Wittgenstein appeals in this part of his text: i.e., the picture of fitting and the picture of belonging? My own view is that two concepts fit each other, but one does not belong to the other, when their ranges are not necessarily related. For example, the concept of redness fits the concept of a rose because some roses are red, but they do not have to be red or not red. The same goes for the concepts of a cat and a being living on Earth: They fit each other because even though all cats live on Earth, there might have been some that did not live there. By contrast, those included in the pairs *proposition* and *truth*, *rule* and *agreement*, and *integer* and *parity* are such that grasping one of the relevant pair requires that we grasp the other. (In the case of some pairs of this sort, the converse relationship also holds.) Wittgenstein says, of concepts that are related in this way, that one belongs to the other. Someone who does not grasp the concept of truth will also not grasp the concept of a proposition, and someone who does not understand the concept of a proposition will also not understand the concept of truth. Likewise, a person who does not grasp the concept of agreement will not grasp the concept of a rule. In the case of two concepts that merely fit each other, grasping one of these concepts is not necessary for understanding the other one. One can possess the

concept of a rose without being in possession of the concept of redness, and vice versa.

So in what way should we seek to elucidate the sense of Wittgenstein's statement that the use of the words "true" and "false" belongs to our concept of a proposition?⁶ One way of explaining what the belonging of the use of these words to the concept of a proposition consists in has already been presented above. According to the *Tractatus*, every proposition can be written in the form of a truth-table; and truth-tables are composed of the symbols "T" and "F," which have the same meaning as the words "true" and "false" (1922, 4.31). Thus, when it comes to expressing propositions in such a notation, the use of these words belongs in a quite literal way to the concept of a proposition. However, some light is shed on what the belonging of the use of the words "true" and "false" to the concept of a proposition consists in by, above all, the following remark from the *Investigations*:

And to say that a proposition is whatever can be true or false amounts to saying: We call something a proposition if *in our language* we apply the calculus of truth functions to it. (Wittgenstein, 2009, § 136)

In this part of the text, Wittgenstein expresses the thought that propositions are something to which we apply truth functions. As is well known, truth functions are those functions whose arguments and values are the truth-values *truth* and *falsity*. This elucidation of the concept of a proposition shows that the latter presupposes the concepts of truth and falsity, and—in that sense—the use of the words "true" and "false" belongs to the concept of a proposition. That is, in order to understand any complex proposition, one has to know how its truth-value depends on the truth-values of its components, and in the case of elementary propositions, one has to know their truth conditions and, thereby, their conditions for being false. Of course, this should not be construed as Wittgenstein's claiming that the concept of a proposition can be fully explained in terms of the concepts of truth and falsity; his point is only that without understanding these concepts, one will not be able to grasp our concept of a proposition (cf. Bronzo, 2019).

⁶ This issue is interestingly discussed by Bartunek (2019).

The Grammar of the Expression “Is True”

In § 137 of the *Investigations*, Wittgenstein qualifies his statement according to which the use of the words “true” and “false” belongs to, but does not fit, the concept of a proposition. He points out that, in some sense, one can recognize that the use of these words fits propositions. He explains what this kind of fitting amounts to as follows:

In *that* sense “true” and “false” could be said to fit propositions; and a child might be taught to distinguish propositions from other expressions by being told “Ask yourself if you can say ‘is true’ after it. If these words fit, it’s a proposition.” (And in the same way one might have said: Ask yourself if you can put the words “*This is how things are:*” in front of it.) (Wittgenstein, 2009, § 137)

The fit he is talking about in this remark consists simply in the fact that if the words “is true” can be said after uttering some expression, these words fit this expression and, in that case, this expression is a proposition. In other words, the expression “is true” fits propositions because if one concatenates any proposition enclosed in quotation marks with this expression, one will obtain a meaningful whole.⁷

The quoted excerpt from § 137 is also important as regards the main topic of our discussion for another reason. In this part of his text, Wittgenstein considers two expressions, “is true” and “*This is how things are:*” and points out that they play a similar role in our language. The possibility of concatenating them with a given expression shows that this expression is a proposition. Of course, there are some differences between their uses. The former can be placed after a proposition; moreover, for Wittgenstein as it seems, it can be put after a proposition that is enclosed in quotation marks. Meanwhile the latter, “*This is how things are:*” can be put before a proposition, and—in addition—before a proposition which is not enclosed in quotation marks. These expressions differ also in the following respect: that from the point of view of ordinary grammar, the former is a predicate,

⁷ The possibility in question is, of course, a logical possibility, and according to Wittgenstein, if it is logically possible that things are thus and so, then the statement that things are thus and so has sense (cf. Wittgenstein, 2009, § 251, § 253, § 520). So, the possibility of ascribing truth to some expression implies that the statement “The expression ‘e’ is true” has sense.

but the latter not. If we take into account the division of expressions into syntactic categories, we can characterize the former as a sentence-forming functor with a name as its argument, and the latter as one with a sentence as its argument. It should also be added that the latter is used as an operator, and plays a similar role to the expression “it is true that.”

In order to better understand the differences between the uses of the expressions “is true,” “*This* is how things are:” and “it is true that,” it is worth recalling again the following remark from the *Investigations*:

And to say that a proposition is whatever can be true or false amounts to saying: We call something a proposition if *in our language* we apply the calculus of truth functions to it. (Wittgenstein, 2009, § 136)

One can say that when we utter a proposition of the form “‘p’ is true,” we *assert that*, in this case, the calculus of truth functions is applied to an expression of the form “p” in the following way:

p	Tp
1	1
0	0

The sign “T” occurring in this truth-table is the truth-connective “it is true that.” By contrast, when we utter a proposition of the form “This is how things are: p,” we are simply *applying* the calculus of truth functions to an expression of the form “p” in the way presented above. This difference in use might be recognized as fundamental, were it not for the fact that Wittgenstein emphasizes that “[t]he proposition “‘p’ is true” can only be understood if one understands the grammar of the sign “p” as a propositional sign” (Wittgenstein, 1978, pp. 123–124). Thus, when we use a proposition of the form “‘p’ is true,” we are not only asserting that, in this case, the calculus of truth functions is applied to an expression of the form “p” in that way, but are also simply applying the calculus of truth functions to an expression of this form in that way; the fact that Wittgenstein recognizes the schema “‘p’ is true = p” as a rule of grammar shows that from his point of view it is an adequate interpretation of the role played in our language by propositions of the form “‘p’ is true.”

So, if one accepts Wittgenstein’s point of view on the sense of propositions of the form “‘p’ is true,” then the differences in use of the expressions

“is true,” “this is how things are:” and “it is true that” can be interpreted as resulting from this: that propositions having the forms “‘p’ is true,” “This is how things are: p,” and “It is true that p” merely emphasize different facets of the same underlying fact—this being that it is true that p—and not from their saying anything fundamentally different from one another. Statements of the first type emphasize, first of all, that we are in the business of asserting how things are *by means of propositions*, whereas statements of the second and third types stress the idea that when we say that it is true that things are thus and so, we are saying *how things are*.

Even so, the fact that different aspects can be emphasized by propositions of those kinds does not mean that, for Wittgenstein, the uses of, for example, the following propositions fundamentally differ from one another:

1. “‘Paris is bigger than Rome’ is true.”
2. “This is how things are: Paris is bigger than Rome.”
3. “It is true that Paris is bigger than Rome.”

It must therefore be acknowledged that the grammatical differences indicated above are not of any fundamental significance.⁸ It turns out that, in some contexts, the concept of truth is expressed by an expression that looks like a predicate—i.e., by the expression “is true”—but which is in fact being used in a different way from ordinary predicates. Since, as I have already pointed out, the use of this expression is similar to the use of the expressions “this is how things are:” and “it is true that,” one can say that from the point of view of grammar in Wittgenstein’s sense of the term, it plays the role of an operator, even though it looks like a predicate. This means that it is not an expression of the kind that we use to ascribe some property to something, and hence that truth is not a property; its use consists in the fact that the result of its being applied to any given proposition is a proposition having the same content as the original proposition.⁹ It is worth emphasizing here that one of the most important ideas of both Wittgenstein’s early and his late philosophy is the conviction that the external similarity of various expressions may hide essential differences in their use:

⁸ Of course, the word “grammatical” is being used here in the linguistic, not the Wittgensteinian sense of the term.

⁹ Prior appears to have drawn similar conclusions later; it should be emphasized, however, that according to him the expression “is true” plays the role of a predicate when predicated of sentences (cf. Prior, 1971).

Of course, what confuses us is the uniform appearance of words when we hear them in speech, or see them written or in print. For their use is not that obvious. Especially when we are doing philosophy! (Wittgenstein, 2009, § 11)

Applying this general observation to the issue under discussion here leads to the conclusion that the external similarity of the expression “is true” to ordinary predicates in no way entails that it need actually play the role of a predicate. As Wittgenstein points out, its use differs fundamentally from the use of ordinary predicates (cf. Diamond, 2003).

A Wittgensteinian Perspective on the Problem of Truth-Bearers and the Redundancy Conception of Truth

The considerations above regarding Wittgenstein’s remarks on truth raise two further issues: firstly, the question of how Wittgenstein approaches the problem of truth-bearers, and secondly, that of whether he ought to be considered an adherent of the redundancy conception of truth. (The second issue is discussed by several commentators (see Baker & Hacker, 2005, Bartunek, 2019, Diamond, 2003, Horwich, 2016, McFarland, 2020, Vision, 2005)). My attempt to answer these issues will, in a way, be tantamount to giving a summary of my preferred interpretation of paragraphs 134–137 of *Philosophical Investigations*.

The issue of so-called “truth-bearers” is often treated as one of the most important problems connected with the concept of truth (Künne, 2003, pp. 15–16). Put very briefly, it is about what kind of entities the property of being true should be attributed to. Some philosophers argue that the property of being true belongs to propositions (Horwich, 1990, Künne, 2003), others that it belongs to statements (cf. Strawson, 1971), while still other ones claim that it belongs to sentences (cf. Quine, 1986, pp. 10–12, Tarski, 1944), and yet still others claim that it belongs to ordered triples of sentences, times, and persons (see Davidson, 1967). It is noteworthy that the proponents of all these solutions make a common assumption to the effect that truth is a property. The source of this assumption is, ostensibly, the fact that the external form of many statements asserting that it is true that things are thus and so has the form “S is P”—i.e., that of a subject-predicate sentence—and on this basis it is concluded that their mode of employment is

not fundamentally different from that of other subject-predicate sentences, which are used to ascribe properties to objects.

In my opinion, it is this very assumption that is problematic for Wittgenstein. At the same time, it should be emphasized that it would be a mistake to formulate his objections to it as amounting to the claim that the property of being true does not exist. To properly understand the essential point of his concerns and reservations, it can be helpful to recall those remarks in which he seeks to explain the real nature of his criticisms of various philosophical claims and positions:

If I speak of a fiction, then it is of a grammatical fiction. (Wittgenstein, 2009, § 307)

The decisive movement in the conjuring trick has been made, and it was the very one that seemed to us quite innocent. (Wittgenstein, 2009, § 308)

The object of Wittgenstein's criticism being considered in these comments is a certain sort of grammatical fiction that consists in misunderstanding the grammar of those propositions that say something is true. The fiction consists in construing the use of those sentences as substantially similar to the use of other subject-predicate sentences. It is treating propositions of the form "'p' is true" in this manner that is "[t]he decisive movement in the conjuring trick," and this movement seems quite innocent to us. So what does Wittgenstein's rejection of this grammatical fiction really signify? As I have already said, it does not amount to the claim that the property of being true does not exist. Rather, it should be expressed in the following terms: The expression "is true" is not being used as a genuine predicate, and for this reason the question "To what property does this expression refer to?" has no sense.

What, then, are the consequences for the problem of so-called "truth-bearers" of rejecting this assumption? In my judgement, there are two possible alternatives when the assumption is to be dispensed with. Where the first of these is concerned, it needs to be demonstrated that the question "To what kind of entities should the property of being true be attributed?" is meaningless. As to the second, the problem of truth-bearers somehow needs to be reformulated. That this question is meaningless can be easily seen if one realizes that while the expression "is true" is, from the point of view of standard grammar, a predicate, it is used rather as an operator

than as an ordinary predicate. This interpretation of Wittgenstein's approach to the problem of truth-bearers is supported by the following quotation:

The words “true” and “false” are two words on which philosophy has turned, and it is very important to see that philosophy always turns upon nonsensical questions. (Wittgenstein, 2001, p. 106)

Could the issue of truth-bearers be formulated in some other way, so that it would make sense? In my opinion, the answer is “yes,” and the reformulated question might run something like this: By what means do we assert (or express) the truth? In principle, the truth is asserted (or expressed) by means of meaningful sentences, i.e., propositions, although in some cases it is asserted (or expressed) by means of such expressions as, for example, “yes” and “no.” Thus, the solution to the problem of truth-bearers so construed turns out to be quite trivial, and this fact is in line with Wittgenstein's remark that “[i]f someone were to advance *theses* in philosophy, it would never be possible to debate them, because everyone would agree to them” (Wittgenstein, 2009, § 128).

I shall now turn to the second question to be addressed here, i.e., that of whether we should regard Wittgenstein as an adherent of the redundancy conception of truth. To put the matter briefly, according to the latter conception the concept of truth is redundant and, in fact, does not play any vital role at all when it comes to understanding our language.¹⁰ This is often explained in the following terms: All sentences with the expressions “is true” and “it is true that” can be substituted—without any loss of sense—with sentences in which these expressions do not occur. Wittgenstein, as it seems, does not endorse the first statement, which says that the concept of truth is redundant (cf. Diamond, 2003), but he does—seemingly paradoxically—agree with the second, to the effect that one can do away with the phrases “is true” and “it is true that” altogether. The fact that he agrees with the latter is evidenced by the following passages:

‘p’ is true = p
‘p’ is false = not-p. (Wittgenstein, 2009, § 136)

For what does a proposition's ‘being true’ mean? ‘p’ is true = p.
(That is the answer) (Wittgenstein, 1991, p. 117 (Appendix III, § 6))

¹⁰ Ramsey (1927) is considered to be the originator of the redundancy conception of truth.

Discussion of these words is made easier once it is realized that the words “true” and “false” can be done away with altogether. Instead of saying “p is true” we shall say “p,” and instead of “p is false” we shall say, “not-p.” (Wittgenstein, 2001, p. 106)

What the above quotations clearly show is that, for Wittgenstein, the phrase “is true” can be eliminated from our language without diminishing the latter’s expressive resources. This does not mean, however, that he views the concept of truth as redundant, or as not playing any significant role in our language. This more nuanced interpretation of Wittgenstein’s approach to the concept of truth is well supported by the following passages:

And the use of the words “true” and “false” may also be a constituent part of this game; and we treat it as belonging to our concept ‘proposition,’ but it doesn’t ‘fit’. (Wittgenstein, 2009, § 136)

That is, instead of the notions of truth and falsity, we use proposition and negation. That we can do this is a useful hint, but it does do away with the puzzles connected with truth and falsity.*

*On the other hand, we could do away with negation, disjunction, conjunction, etc., and use true and false, making up a notation containing only the words “true” and “false”. (Wittgenstein, 2001, p. 106)

Wittgenstein recognizes, firstly, that the use of the words “true” and “false” belongs to the concept of a proposition, such that it is impossible to fully explain what a proposition is without invoking the concepts of truth and falsity.¹¹ Secondly, in his view the fact that one can use the notions of truth and falsity instead of the notions of proposition and negation does not allow us to avoid “the puzzles connected with truth and falsity.” And finally, thirdly, Wittgenstein points out that a kind of reverse operation can be performed. That is to say, instead of using the notions of negation, conjunction, etc., one can use just the notions of truth and falsity.¹² On the basis of these three points it must be concluded, I think, that according to Wittgenstein

¹¹ The interpretation presented here is incompatible with those readings of Wittgenstein according to which “the concept of truth has no additional normative content beyond that which the notion of warranted assertibility, or justifiability according to socially accepted standards, has of its own” (Frascolla, 2017, p. 215).

¹² How this can be done had already been outlined by him in the *Tractatus* (cf. Wittgenstein, 1922, 4.442).

the concepts of truth and falsity cannot be explained in terms of a prior understanding of the notions of a proposition and its negation. Here, the expression “prior understanding” means no more and no less than such an understanding as does not itself refer to the concepts of truth and falsity (cf. Bronzo, 2019). In other words, for Wittgenstein, neither of the concepts from the pair consisting of that of a proposition and that of truth is more basic than the other—neither of these concepts is prior to the other one.

To sum up, if the redundancy conception of truth boils down to just the claims that (1) “‘p’ is true = p” is a rule of grammar in the Wittgensteinian sense of the term and (2) the phrase “is true” can be eliminated from our language without diminishing its means of expression, then Wittgenstein’s remarks on truth may be considered to be in line with it. However, if this conception assumes that the concept of truth is actually redundant and can be fully explained in terms of some more basic concepts—for example, in terms of the concept of a proposition or the concept of the content of a proposition—then Wittgenstein’s approach to truth cannot be so construed.

What Is the Difference between the Investigations’ Approach to Truth and That of the *Tractatus*?

Is there any significant novelty in the remarks on truth in paragraphs 134–137 of the *Investigations* compared to the *Tractatus*’ account of truth? The answer to this question is, “Yes.” The main source of this novelty is a different approach to the issue of the general form of the proposition. According to the *Tractatus*, since the “[g]eneral form of proposition is: Such and such is the case” (Wittgenstein, 1922, 4.5), the concepts of truth and falsity belong to every proposition (cf. Wittgenstein, 1922, 6.111). On the other hand, Wittgenstein in the *Investigations* recognizes that the search for the general form of the proposition is completely futile (cf. 2009, § 65), and that the concept of a proposition is a family-resemblance concept—“these phenomena have no one thing in common in virtue of which we use the same word for all—but there are many different kinds of *affinity* between them” (2009, § 65). Therefore, in the *Investigations* Wittgenstein does not claim that the concepts of truth and falsehood *simply* belong to the concept of a proposition, but he points out that in *many cases* when we use propositions, the use of the words “true” and “false” belong to the use of these propositions. Whether

the use of these words belongs to propositions or not depends on what language-game we are playing. When we take into account such a game as that presented in § 8 of the *Investigations*, i.e., the game of giving orders like “fourth-slab-there,” “this-there,” we may be inclined to consider such utterances as propositions, but the concepts of truth and falsehood will not apply to the propositions belonging to this language-game. On the other hand, these concepts would undoubtedly be applied to the propositions belonging to the game which would consist in describing the activities of the participants of the above-mentioned language-game; these concepts can also be applied to the propositions belonging to the game presented in § 48 of the *Investigations*. To sum up, since in the *Investigations* Wittgenstein rejects the explanation of the essence of a proposition in terms of its general form and points out that whether or not the concept of truth belongs to a given proposition depends on how that proposition is used in a given language-game, he recognizes that to clarify the concepts of truth and proposition it is necessary to grasp the similarities and differences between those language-games that include the use of the words “true” and “false” and those that do not. However, as I have already mentioned, the author of the *Investigations* agrees with the author of the *Tractatus*, that basically the concept of truth and the concept of a proposition are closely related.

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Towards a Computational Ontology for the Philosophy of Wittgenstein: Representing Aspects of the *Tractarian* Philosophy of Mathematics

Keywords: Ludwig Wittgenstein, Semantic Web, Knowledge Representation, *Tractatus Logico-Philosophicus*, Philosophy of Mathematics

Słowa kluczowe: Ludwig Wittgenstein, sieć semantyczna, reprezentacja wiedzy, *Tractatus logico-philosophicus*, filozofia matematyki

Abstract

The present paper concerns the Wittgenstein ontology project: an attempt to create a Semantic Web representation of Ludwig Wittgenstein's philosophy. The project has been in development since 2006, and its current state enables users to search for information about Wittgenstein-related documents and the documents themselves. However, the developers have much more ambitious goals: they attempt to provide a philosophical subject matter knowledge base that would comprise the claims and concepts formulated by the philosopher. The current knowledge representation technology is not well-suited for this task, and a non-standard approach is required. The creators of the Wittgenstein ontology project are aware of this fact; recently, they have been discussing conceptual devices adjusting the technology to their needs.

The main goal of this paper is to present examples of a representation of philosophical content that make use of both the devices already proposed and some new

inventions. The represented content comes from the *Tractatus Logico-Philosophicus*; more specifically, its theses concerning the problems in philosophy of mathematics.

Introduction

The Wittgenstein ontology project is a unique interdisciplinary research initiative that links philosophy and information science. The goal of the project is to create a Wittgenstein-related knowledge base that would be accessible to relatively simple algorithmic systems and easily searchable by users. The project has been run by the Wittgenstein Archives at the University of Bergen, Norway (WAB) since 2006. So far, the WAB team has been able to complete the part of the base that covers information about Wittgenstein's published and unpublished texts as well as their internal structure. They have also published a couple of theoretical papers regarding the possibility of representing philosophical content.

The present paper discusses an attempt at putting their ideas into practice. More specifically, it focuses on the segment of the *Tractatus Logico-Philosophicus* devoted to the philosophy of mathematics. There are several reasons why this is a much more difficult task than developing a knowledge base concerning just documents. The proposed solutions are sometimes quite complicated. Nevertheless, I believe that they demonstrate that the task is generally feasible.

The paper begins with a very short introduction to the paradigm being used in the development of the Wittgenstein ontology project: the Semantic Web technology. Next, we proceed to discuss the WAB's practical and theoretical achievements: both the current state of the project and its visions and proposals. However, the main goal of this paper is to propel the Wittgenstein ontology project forward and demonstrate how we can actually create representations of philosophical knowledge using the approach of the WAB researchers.

Our case study will be a fragment of the *Tractatus Logico-Philosophicus*; more precisely, theses 6–6.031 and 6.2–6.241. However, a full representation of this part of the philosophy of the early Wittgenstein, due to limitations of space and the relatively large scale and complexity of the task, will not be presented here; we shall only discuss several interesting sentences. I believe that although the reader is acquainted with only a limited

number of examples, they will be capable of imagining the further development of the Wittgenstein ontology project.

Basic information on the technology of knowledge representation

The Semantic Web technology (SW) is a paradigm within Knowledge Representation & Reasoning (KR²), a field in artificial intelligence research. SW enables us to represent and share information that can be easily searched, retrieved, and processed by both human users and automatic agents. Basic units of SW information are uniquely identifiable resources that can be assigned various primitive alphanumeric values and linked together through binary relations, thus forming complex knowledge graphs. Generally, there are three types of such resources: entities, object properties, and data properties. Entities can only occur as nodes in a graph; they should be considered objects that can be predicated with various values. They can be grouped into so-called classes (one entity can belong to more than one class). Object properties represent binary relations between entities; more precisely, an object property can be attributed to an entity and assign to that entity a different resource as a value. In turn, data properties are also attributable to entities, but their values are alphanumeric strings rather than resources. Both object and data properties can have certain meta-properties, like domain, range, and being functional (attributable only once to a single entity). Additionally, object properties can have such meta-properties as symmetry, transitivity, and reflexivity.

Resource Description Framework (RDF) is a method or language developed by the World Wide Web Consortium (W3C) that codifies the most general principles of representing knowledge according to the SW paradigm. Together with RDF Schema and Web Ontology Language (OWL), which allow for the creation of hierarchies of classes and properties, and SPARQL, a query language used to retrieve and manipulate information stored in SW knowledge bases, it provides the foundations for the technology under discussion. Within an SW knowledge base, the information is chunked into so-called RDF triples: each triple is articulated in the subject, property, and object. The subject can be a resource (an entity or a data/object property); the middle member of a triple can be data or object property; and the object can be a resource or an alphanumeric value. The structures that comprise the

RDF triples are called named graphs. They can be presented in graphic form or stored in text files coded in one of several serialization formats (N-Triples, N-Quads, Turtle, JSON-DL, RDF/XML, and so on). The “name” of a graph is its unique identifier; typically, its URL address.

Although the subject-property-object triple structure seems too limited to express a variety of propositional forms, a couple of RDF syntax additional features broaden the expressiveness of name graphs. One of them is the so-called blank node: a pseudo-entity that can occur in the place of a subject or an object within a triple without being a resource, namely, without having any identifier. Though it can have a name that identifies it locally within a given named graph, it can occur unnamed as well. Blank nodes are useful when we try to represent such propositions as: “John goes to Kirchberg by car.” We would try to represent this example with two triples: “John”—“goes to”—“Kirchberg” and “John”—“goes by”—“car.” However, such a representation would not inform us that this particular John’s trip to Kirchberg is made by car. Therefore, it would be better to use a blank node and produce three triples: “John”—“travels”—[blank node X], [blank node X]—“to”—“Kirchberg,” and [blank node X]—“by”—“car.” Here, the blank node clearly represents the travel itself that is not explicitly mentioned in the sentence. The situation is presented graphically in Diagram 1.

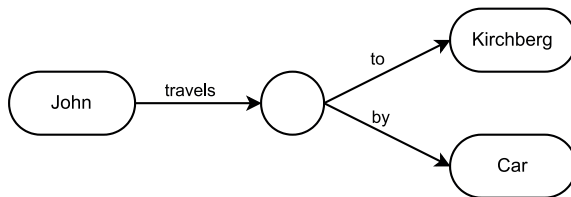


Diagram 1. The named ovals symbolize the entities (nodes); the arrows mark the properties (they point to their values); and the empty circle represents the blank node

Another useful RDF feature is reification, which allows for the representation of a sentence (in fact, an RDF triple) as an object. The object in question (which can be either a resource or a blank node) belongs to the special predefined RDF class “Statement.” A “Statement” object has three special object properties: “Subject,” “Predicate,” and “Object” that have, as

their values, the respective members of a triple that is to be reified. The reification mechanism allows us to create representations of intensional contexts such as “Mary believes that John is in Kirchberg”: “Mary”—“believes”—[blank node R], [blank node R]—“type”—“Statement,” [blank node R]—“Subject”—“John,” [blank node R]—“Predicate”—“is in,” [blank node R]—“Object”—“Kirchberg.” Certainly, there is no direct assertion attributed to the reified part: only the triples can assert anything within an RDF knowledge base. As we can see, the reification syntax is quite complicated (the example discussed above is presented in Diagram 2), but it is inevitable for various purposes that are pertinent to the goal of this paper.

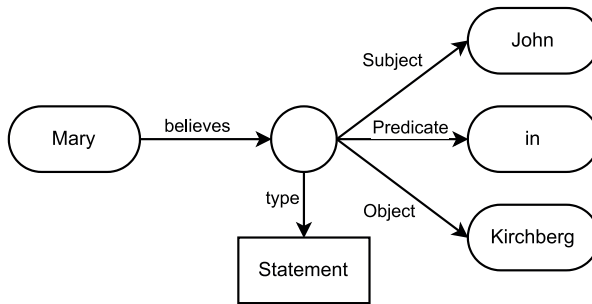


Diagram 2. The rectangular “Statement” node is a class: a group of entities of the same type; the reader should note that the property “in” occurs as a node (a value of the object property “Predicate”)

WAB’s Solutions to the Challenges of Representing Philosophical Knowledge

The present state of the SW paradigm facilitates its application to a specific kind of knowledge: well-established, clear-cut, non-debatable. Similar to information about a collection of documents, we can represent volumes, single pages, and even paragraphs; we can link them to people, topics, dates, and other documents.¹ We always know to which category a given entity

¹ Other suitable examples can be uncontroversial areas of science such as anatomy and genomics (Rosse & Mejino, 2003; Smith, Köhler & Kumar, 2004; see also Pichler et al., 2021, p. 62).

should be assigned; we know all the relationships between a single entity and others. An example of such an SW knowledge base is the working part of the Wittgenstein ontology project under development by the WAB. The interface of its explorer is available at the following web address: <http://wab.uib.no/sfb>. In its current form, the project is successful as an automatic index that covers the entire Wittgenstein *Nachlass* and all his published works. It should be underlined that the basic entities that the knowledge base consists of are not separate pages but single remarks that Wittgenstein sometimes numbered and often dated; there are over sixty thousand of them.

The KR²-friendly knowledge, as was noticed by the group of authors related to the WAB in their 2021 paper “Crisscross Ontology” (Pichler et al., 2021), is characterized by stability, precision, and coherence. The task of representing such knowledge can be compared to arranging jigsaw puzzles: it consumes a significant amount of time, but eventually all the pieces find their right places. Obviously, not all our knowledge can be described in this way. Quite often, especially in science, our knowledge is very far from being precise and stable; there are systematically vague concepts, dynamisms, tensions, and conflicting interpretations. The situation in philosophy is even worse: as has been observed by Ludwig Wittgenstein, the very nature of philosophical investigation prevents us from proceeding along a single track of reasoning and “compels us to travel crisscross in every direction over a wide field of thought” (Wittgenstein, 2009, p. 3). Therefore, the WAB scholars use the term “crisscross” when referring to knowledge that is characterized by vagueness, instability, and multiperspectivism (Pichler et al., 2021, pp. 59–60). It seems that, like in David Chalmers’ vision of the philosophy of mind, there are easy and hard problems within the KR² domain as well: the task of representing philosophical knowledge appears to be significantly more challenging than creating a knowledge base for a collection of some writings. Nevertheless, this is precisely the main goal of the Wittgenstein ontology project.

The WAB team is fully aware of the difficulty of the task they have undertaken. After trying out some misleading methods based on the apparent structural similarities of formal computational ontologies and philosophical ontologies such as that apparently presented by the *Tractatus Logico-Philosophicus* (Zöllner-Weber & Pichler, 2007), they have become convinced that a new model of knowledge representation is needed (Mácha,

Falch, & Pichler, 2013; Pichler et al., 2021, p. 71). So far, their research has resulted in proposing a flat conceptual structure whose central elements are the three classes: “Perspective,” “Claim” (or “Point”), and “Concept” (or “Issue”). The alternative names for the latter two classes given in parentheses occur in the most recent version of the Wittgenstein ontology file (Pichler, Gjesdal & Ruwehy, 2007-, available for download from the following web address: http://wab.uib.no/cost-a32_philospace/wittgenstein.owl), but the WAB researchers have tended to replace them with “Claim” and “Concept” in their recent written (cf. Pichler et al., 2021, p. 62) and spoken statements. However, so far, the proposals the WAB team offers are rather inchoate and theoretical. Although the 2021 paper attempts to outline the relationships between possible members of the three classes, it ends with the caveat that the solutions are not yet definite and sufficiently detailed (Pichler et al., 2021, p. 71). They do not provide any real examples, either.

The Wittgenstein Ontology Class Structure

Before we proceed to the representation of the *Tractatus*’ philosophical content, we need to discuss some issues related to the project’s class hierarchy. Its current version includes the two main branches: the first of them, rooted in the “Source” main class, gathers classes responsible for representing Wittgenstein’s remarks, documents, volumes, and publications. The second, meanwhile, subordinated to the “Subject” main class, is designed for dealing with the philosophical subject matter. While the latter contains the three aforementioned classes responsible for perspectives, claims, and concepts, the former includes classes that are the backbone of the working part of the project. Among them, the two most important are “Nachlass Bemerkung” and “Part”: these are groups of entities that represent particular paragraphs, numbered theses, or remarks (the former is responsible for the unpublished *Nachlass* content, while the latter deals with the published texts). Wider-scope “Source” classes are: “Chapter,” “Werk,” “MS,” and “TS”; these consist of entities representing chapters, published books, *Nachlass* manuscripts, and typescripts, respectively. There is also one narrower-scope class; namely, “Sentence,” which represents single sentences that comprise Wittgenstein’s

texts. It should be mentioned that the “Sentence” class is currently empty; the WAB has not yet filled it with entities.²

Although my general approach to the current Wittgenstein ontology class hierarchy is conservative, I propose a slight extension to it, which would be particularly important for the focal point of the present paper; that is, the Tractarian philosophy of mathematics. In addition to the existing classes, I shall introduce the “Symbol” class that groups representations of logical and mathematical formulas. The function of the “Symbol” nodes will be closely linked to yet another proposed extension of the class hierarchy: the “Ruleset” class that would be responsible for non-standard content-related automated reasoning within the SW knowledge base. Despite providing very interesting possibilities, the “Ruleset” class will not be discussed any further in the present paper. The third additional class—“Clause”—represents conceptual parts of claims that have no assertion. Typically, “Clause” members are the premises and conclusions of conditional expressions. There will be some examples of the “Clause” class application in the course of the paper.

It seems uncontroversial that the place of “Clause” and “Ruleset” is among “Subject” descendants. In turn, one can dispute the place of the “Symbol” class within the hierarchy. On the one hand, it fits in the “Subject” family because its members are involved in relationships with concepts, clauses, claims, and rulesets. On the other, “Symbol” nodes would represent literal parts of the source material and for this reason the class should rather

² Adding “Sentence” nodes would significantly expand the Wittgenstein ontology knowledge base, whose current size is slightly more than 520,000 RDF triples. Assuming that each separate paragraph or remark consists of, on average, three sentences, Wittgenstein’s published and unpublished texts comprise roughly two hundred thousand sentences. Each sentence node should be attributed with at least three properties: “type” with the value of the “Sentence” class; “is part of” with the value of a particular “Part” or “Nachlass Bemerkung” entity that includes a given sentence; and a data property with the value being the actual text of a given sentence. (For now let us put aside the fact that a large part of Wittgenstein’s texts have translations; therefore, the latter property should be attributed more than once to a number of sentence nodes with values in different languages.) This yields at least six hundred thousand new RDF triples; the knowledge base would therefore double its size. Fortunately, having both the *Nachlass* transcriptions and published texts already divided into remarks, we can easily automate generating these triples. It is worth mentioning that the WAB has already suggested a naming convention for sentence nodes in the comment to the “Sentence” class (Pichler, Gjesdal & Ruwehy, 2007).

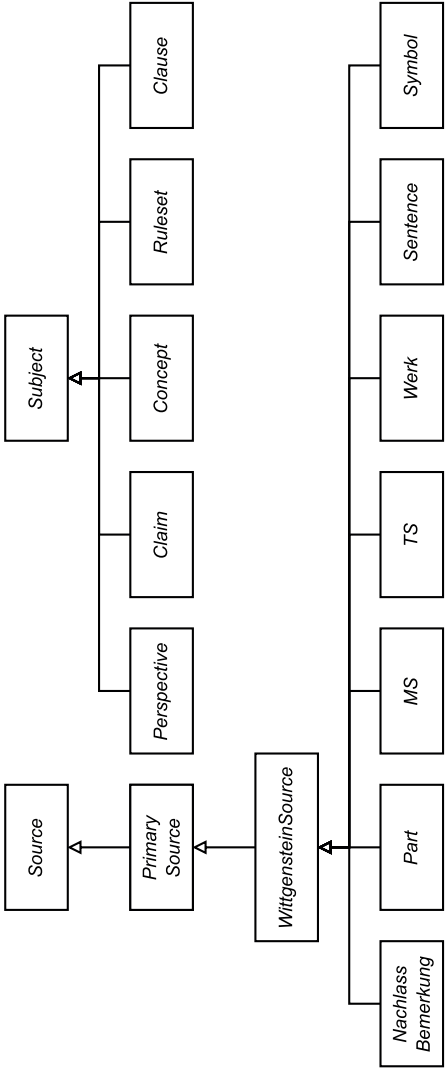


Diagram 3. Fragments of the Wittgenstein ontology class hierarchy (arrows point to parent classes). The “Source” branch is particularly ramified: Both “Primary Source” and “WittgensteinSource” have their siblings (not depicted in the diagram)

be a sibling of “Sentence” and “Nachlass Bemerkung.” Tentatively, I shall take the latter option³ without closing the case. One can also point to another issue related to the “Symbol” class: its name. The early Wittgenstein’s philosophy of logic and language makes use of the concept of symbol and juxtaposes it with the concept of sign. Moreover, it appears that it is the latter part of the juxtaposition that should be applied to logical and mathematical formulas as abstracted from any non-formal contexts of use (cf. e.g., Potter, 2008, p. 210n). Nevertheless, we should refrain from mixing up our conceptual ontology with the categorization that is on the subject matter level. An abridged hierarchy of the Wittgenstein ontology supplemented with the three proposed classes is depicted in Diagram 3.

The SW Technology in Practice

The examples of SW representation of the part of the *Tractatus* presented below are made of instances of the classes: “Sentence,” “Symbol,” “Perspective,” “Claim,” “Concept,” and “Clause.” The graphs are composed according to the following general principles:

- 1) each claim is attributed to a certain perspective; it is also related to both its sentential counterpart (a member of the “Source” branch) and its structure;
- 2) a structure of a claim is a “Statement” instance having all the properties required for the reification mechanism: a claim’s components are values of the properties;
- 3) a structure of a claim can be nested: Both the subject and object of a reified triple can be “Statement” instances themselves; they can also be “Clause,” “Perspective,” or “Claim” instances;
- 4) there are structural meta-properties that are responsible for linking perspectives, sentences, claims, symbols, clauses, rulesets, and statements together; there are also subject matter properties that are components of particular claims.

³ This choice was strongly recommended by Alois Pichler when we discussed the “Symbol” class proposal.

Let us take a look at the representation of the first sentence of the *Tractatus* 6: “The general form of a truth-function is $[p, \zeta, N(\zeta)]$.”⁴ First of all, we need a perspective to anchor the claim; therefore, we introduce the node “Perspective 1” as a subject in the triple: “Perspective 1”—“type”—“Perspective.” Now, we can introduce another node, “Claim TLP 6 [1],” to represent the philosophical claim expressed with the sentence in question. The node appears in four triples: “Claim TLP 6 [1]”—“type”—“Claim,” “Perspective 1”—“claim”—“Claim TLP 6 [1],” “Claim TLP 6 [1]”—“source of claim”—“Tractatus logico-philosophicus (TLP, 1921/1922) 6 [1],” and “Claim TLP 6 [1]”—“structure of claim”—“Statement TLP 6 [1].” As we can see, the meta-properties responsible for linking a perspective with a claim, a claim with a sentence, and a claim with a statement are called “claim,” “source of claim,” and “structure of claim,” respectively.⁵ There are two other nodes that occur in the above triples: the “Tractatus logico-philosophicus (TLP, 1921/1922) 6 [1]” node is of a type “Sentence” and is linked to the “Tractatus logico-philosophicus (TLP, 1921/1922) 6” node of a type “Part” (that node is currently present in the knowledge base). It is attributed with a data meta-property “content” that brings its actual content quoted above (as we shall see, the same meta-property is used to attribute the actual formula to “Symbol” instances). In turn, the “Statement TLP 6 [1]” node is of a type “Statement” and is attributed with the three standard reification properties thus occurring in the three following triples: “Statement TLP 6 [1]”—“Subject”—“General form of a truth-function,” “Statement TLP 6 [1]”—“Predicate”—“expressed with a sign,” and “Statement TLP 6 [1]”—“Object”—“Symbol TLP 6 [1].” The “General form of a truth-function” node is a “Concept,” while the “Symbol TLP 6 [1]” node is an instance of the “Symbol” class.

How should we ascribe the members of reified triples to their classes? There are two possible ways: either we could make a context-free ascription, or we could ascribe an entity to a class within a certain perspective as yet another reified triple. Tentatively, let us take the former option; it will make the graphs much less complicated. However, we should be aware that sticking

⁴ All the quotations from the *Tractatus Logico-Philosophicus* come from Pears & McGuinness’s translation (Wittgenstein, 1965).

⁵ By convention, the names of properties are written in lowercase; the names of the three reification properties are the exceptions to this rule.

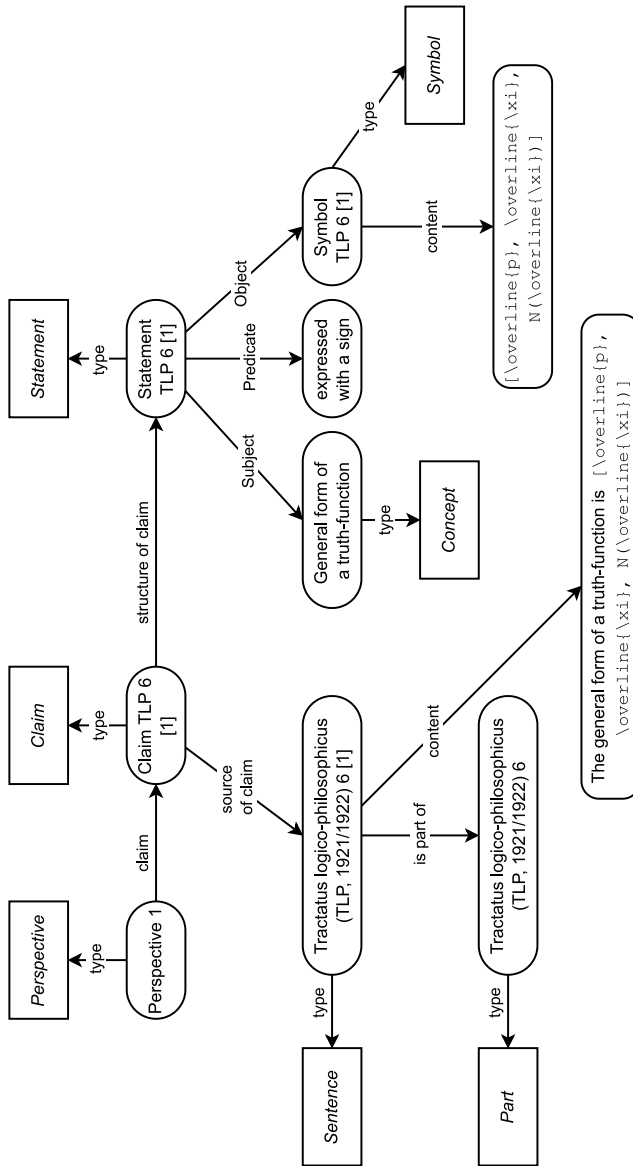


Diagram 4. Rectangular nodes are classes; oval nodes are instances of classes; and rounded rectangles are strings (the values of data properties). The last of these include TEX notation to represent the formula “ $\overline{p}, \overline{N(\overline{xi})}$ ”; the “expressed with a sign” node is not assigned to any class: it is an object property that occurs within a reified triple

to this practice can cause us problems in the future: there can be a disagreement between two perspectives about treating a particular part of a given Wittgenstein's sentence as a concept. In such situations, we should instead follow the latter option and represent an ascription as a reified triple.

The graph for the sentence under discussion is shown in Diagram 4. It is a rather complicated structure compared to the syntactic simplicity of the sentence. After all, we have a complex concept to the left, a complex symbol to the right, and a copula that links them together. Meanwhile, the graph is made of twelve triples (plus four triples about the "Sentence" class member). However, in order to make our presentation more concise, we can transform it to a much simpler structure by "resurfacing" our reified triples; that is, by presenting them as if they had not been subject to reification. Diagram 5 demonstrates the first sentence of the *Tractatus* 6 "resurfaced."

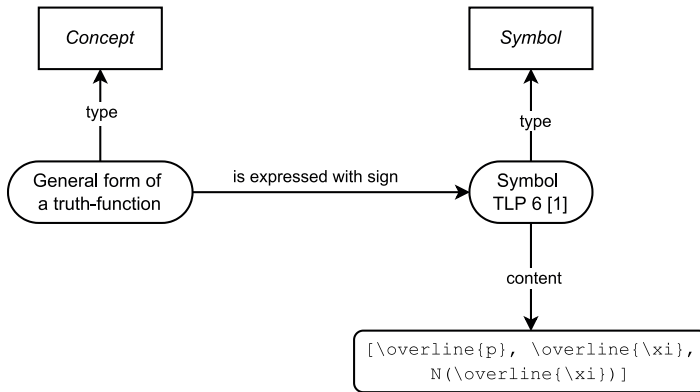


Diagram 5. The triple that represents the first sentence of the *Tractatus* 6

Finally, we come to a true philosophical issue: the meaning of the copula "is." There are two reasons why an SW representation of philosophical content cannot be a simple dissolution of its linguistic form. Firstly, what we call "philosophical content" is a conceptual structure that finds its expression in utterances but does not boil down to them. Sometimes, two separate words are used to express the same conceptual content; likewise, the same word can be used in a couple of different meanings. Secondly, a usable SW knowledge base must have a controlled vocabulary. In other words, the

number of resources must be limited if we want to be capable of making effective searches. This is particularly true for object properties.

Our object property that stands for the linguistic “is” is called “is expressed with sign.” This is just a name for an RDF resource and can be replaced by anything; for example, “OP-TLP-6-1.” However, the crucial matter is in what triple the same property is used again and in what triple it is not. For instance, it does not occur in the representation of the second sentence of the *Tractatus* 6, despite the fact that the latter says, “This *is* the general form of a proposition” (italics mine). The structure of the relevant triple (“resurfaced”) is shown in Diagram 6.

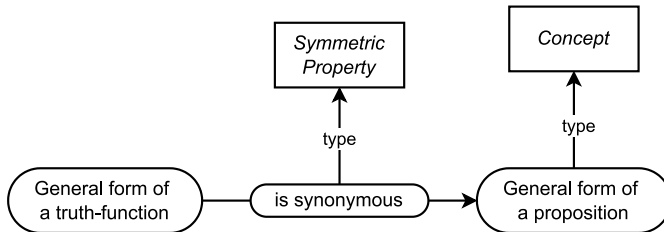


Diagram 6. The triple that represents the second sentence of the *Tractatus* 6.

The main object property “is synonymous” is marked by an oval because the graph also represents its being a member of a certain class of properties

Here we have “is synonymous” in place of the copula. Again, it is just a name that can be replaced by, for instance, “OP-TLP-6-2.” However, the point is that this is a different property than the property “is expressed with sign” and, unlike the latter, it is symmetric. I have also “unwrapped” the anaphoric reference to the previous sentence by putting the right concept instead of the pronoun. The reason why I read “A general form of a truth-function is general form of a proposition” in this way is that the *Tractatus* demonstrates that a proposition is a truth-function.⁶

⁶ One can notice that, according to the *Tractatus*, the concept of proposition is rather explicated by the concept of truth-function than synonymous with it (cf. Wittgenstein, 1965, p. 5). However, I believe that, in general, a relationship between complex concepts can be different from a relationship between their crucial differentiating components. Moreover, the concept of general form of a truth-function can be replaceable with the concept of general form of a proposition exactly for the reason that the

There are two other places in the *Tractatus* 6–6.031 and 6.2–6.241 where we make use of the “expressed with a sign” property: the first sentence of 6.01 and the first (and only) sentence of 6.03. Both ascribe symbols to concepts, and both are translations of the copula “is.” In fact, both are structurally very similar to the first sentence of the *Tractatus* 6. Let us take a closer look at the 6.01 example that is shown in Diagram 7.

The *Tractatus* sentence represented in the graph above is an example of a situation in which a single sentence corresponds to more than one claim. The additional content is placed in parentheses and states that the symbol “[$\bar{\xi}$, N($\bar{\xi}$)]($\bar{\eta}$)” is equivalent to the symbol “[$\bar{\eta}$, $\bar{\xi}$, N($\bar{\xi}$)].” As we can see, the graph is rather complicated, consisting of twenty four triples.

There can also be a situation in which two or more reification statements correspond to a single claim. An example of such a situation is the *Tractatus* 6.001: “What this says is just that every proposition is a result of successive applications to elementary propositions of the operation N($\bar{\xi}$).” This thesis comprises only one sentence, and it makes only one claim. However, the structure of the claim is too complicated to be represented by just one triple. Diagram 8 shows the graph for that part of the *Tractatus*.

The graph includes yet another resolution of an anaphoric reference: in the beginning phrase (“What *this* says...”), the pronoun clearly points to the symbol “[p , $\bar{\xi}$, N($\bar{\xi}$)],” but we only know that from the context of the rest of the sentence, not from its grammatical structure. It happens that the anaphor is ambiguous, and the way we resolve it sometimes depends on our reading of the text. Fortunately, this is not the case here, but ambiguous pronouns do occur in the text of the *Tractatus*.

The second claim of the *Tractatus* 6.001 also includes the copula “is.” Here, like in a couple of other places—6.01 and 6.021—it is represented by the property “explicated as.” So far, this is the third reading of the copula that has been discussed, but we should also point to yet another property used to render it: “is a kind of.” The latter appears in 6.2 and 6.235 (“Mathematics”—“is a kind of”—“Logical method”),⁷ as well as in 6.24 (“Method by which

concept of proposition is explicated by the concept of truth-function: the explication that occurs on a lower level results in synonymy on a higher level.

⁷ It should be pointed out that the first sentence of the *Tractatus* 6.2 and the only sentence of the *Tractatus* 6.235 are represented by the identical reified triple because the differences between the two sentences are only superficial. However, instead of linking the same claim with two source sentences, we create two separate claims and two

mathematics arrives at its equations”—“is a kind of”—“Method of substitution”). Thus, we have four different representations of “is” in our knowledge base: the first of them, “is expressed with sign,” signifies the relation of a given concept being expressed by a given symbol; the second, “is synonymous,” means that two concepts have the same meaning and are interchangeable; the third, “explicated as,” shows that an object-concept brings more information about the meaning of a given subject-concept. Finally, the fourth, “is a kind of,” informs that a subject-concept is a specific type of some broader concept given as a value.

Let us move to yet another interesting cluster of examples. In the first sentence of thesis 6.232, Wittgenstein makes a critical comment about a certain alleged view of Gottlob Frege: “Frege says, that the two expressions have the same meaning but different senses.” This sentence includes an anaphoric reference that is not immediately visible: the phrase “the two expressions” refers to one of the previous theses; namely, the first sentence of thesis 6.23 dealing with two expressions connected with an equals sign: “If two expressions are combined by means of the sign of equality, that means that they can be substituted for one another.”⁸

Let us begin with the latter sentence, which is an example of a conditional statement. First of all, we should resist the temptation to represent it as a FOL rule: this would be as equally futile as trying to construe a philosophical ontology as a computational ontology. Instead, we make use of an object property “entail” that should also be used for similar examples for other in-text inferences.⁹ The property links two “Clause” instances that represent the premise and conclusion. Neither of the two are easy to represent. After resolving the anaphoric reference, which is a non-trivial step, we can obtain the following two clauses: “The expression x is combined with the expression y by the sign of equality” and “The expression x can be substituted by the expression y , and vice versa.” We are forced to use variables because both clauses deal with the same pair of expressions. Hence, we need some formal device to introduce variables to our representation. We can do this

statements. This enables some possible future cross-perspectival links with standpoints that interpret the two sentences differently.

⁸ The reason why I believe that the reference points to thesis 6.23 rather than to 6.231, which is the direct predecessor of thesis 6.232, is that the basic structure of the *Tractatus* is a tree (cf. Stern, 2019).

⁹ Later, such examples will be handled by a mechanism related to the “Ruleset” class.

by means of yet another meta-property that can be called “variable of”; a concept variable will be attributed this property with a value of a concept being the range of the variable. For simplicity’s sake, we shall not introduce any mechanism of limiting the scope of variables; therefore, all the variables within a given knowledge base will be global. We should keep this in mind and use naming conventions to make an order. Therefore, the concept variables in 6.23 will be named “Expression (TLP 6.23 [1]) variable x” and “Expression (TLP 6.23 [1]) variable y.” The subsumption of variables to their concepts can happen outside of the mechanism of reification.

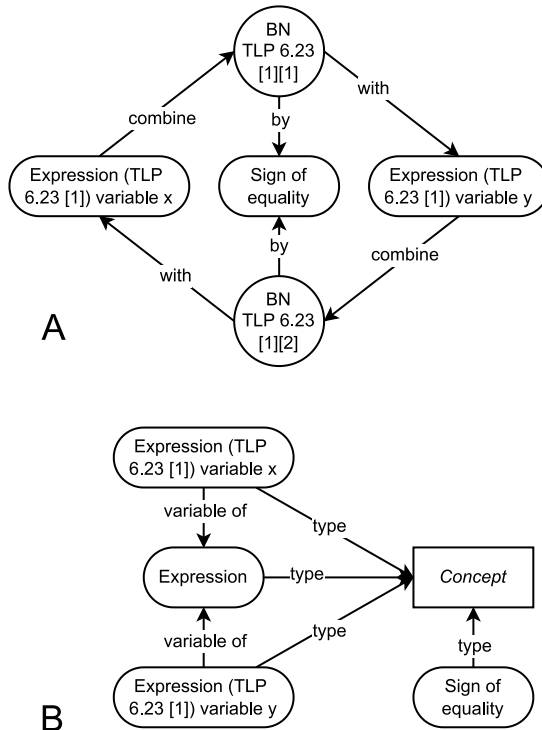


Diagram 9. Figure A is a “resurfaced” part of the graph that will be represented as reified; figure B represents the triples that can occur outside of the reification syntax. Blank nodes (named) are represented as circles

The premise poses another question: how to represent its structure? A less complicated solution is to make a simple triple: “Expression (TLP 6.23 [1]) variable x”—“combined by equals sign”—“Expression (TLP 6.23 [1]) variable y.” There are two shortcomings of such an approach, however: firstly, a very specific object property is added to the vocabulary; secondly, the sign of equality does not figure among the concepts that occur in the claim. Instead, we can take a different approach: use a blank node as a value of the simpler property “combine” together with the two auxiliary properties “by” and “with.” One weakness of this version is that we cannot simply make “combine” symmetric; to represent the symmetry of the relationship between the two variables, we have to double the graph and swap them. Diagram 9 depicts the complete solution of the premise.

Now, we need to subsume the whole graph to one “Clause” entity that would represent the premise. We cannot subsume it to the claim directly because a premise in a conditional statement is not asserted. All six triples that link both expression variables, two blank nodes, and the “Sign of equality” concept are reified, and the “Statement” entities are attached to the “Clause” member called “Premise TLP 6.23 [1]” by the object property “structure of clause.” The whole situation is shown in Diagram 10.

The graph consists of thirty-one triples. It is rather complicated, and yet it is not connected to any perspective. However, since we assume that the “Clause” elements lack assertion, there is no need to anchor them directly in any “Perspective” instance: they will be free-floating “global” objects within our knowledge base. Similarly, the conclusion of the first sentence of thesis 6.23 is such a “global” object; however, it is much less complicated, as can be seen in Diagram 11.

Having the premise and conclusion, we are ready to represent the sentence in question as a reified triple “Premise TLP 6.23 [1]”—“entail”—“Conclusion TLP 6.23 [1].” Diagram 12 depicts the relevant graph.

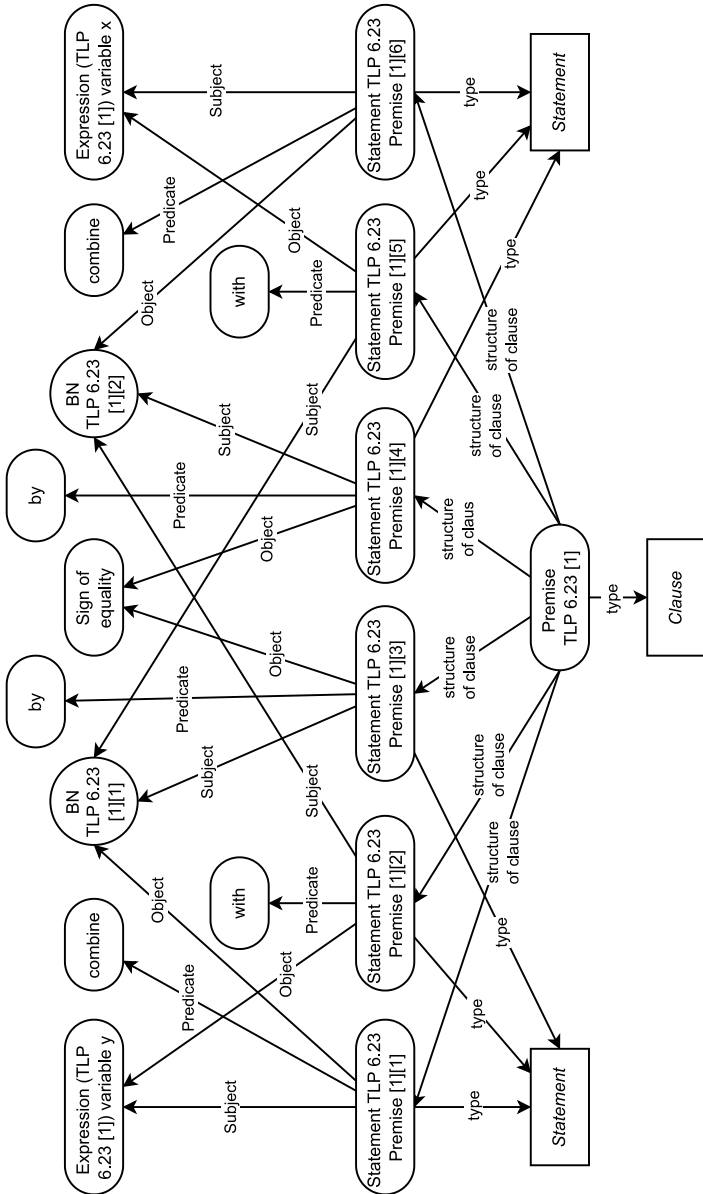


Diagram 10. The premise of the first sentence of the *Tractatus* 6.23. The nodes “combine,” “with,” “by” (properties), and “Statement” (class) are doubled to make the graph easier to read

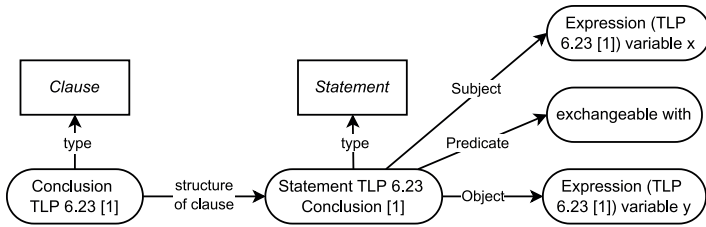


Diagram 11. The conclusion of the first sentence of the *Tractatus* 6.23

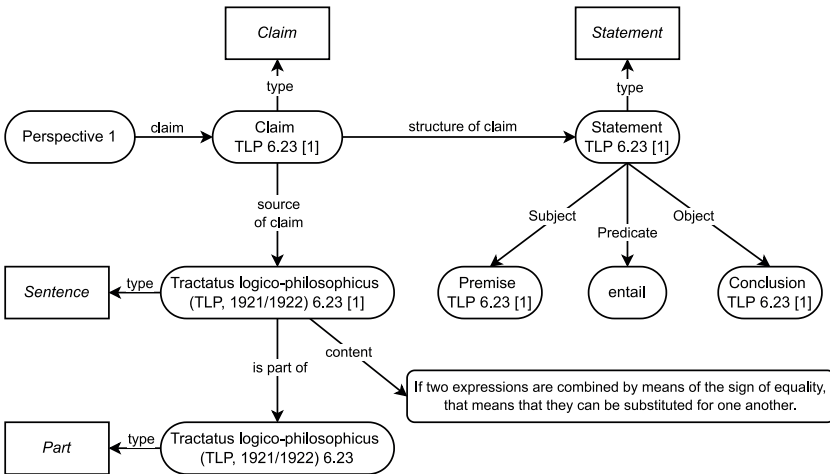


Diagram 12. The general structure related to the first sentence of the *Tractatus* 6.23

Now we can come back to the first sentence of thesis 6.232: “Frege says that the two expressions have the same meaning but different senses.” We know now that the expressions are in fact the two expression variables we have dealt with; that is, “Expression (TLP 6.23 [1]) variable x” and “Expression (TLP 6.23 [1]) variable y”; Wittgenstein says that Frege says that they have the same meaning but different senses. In other words, according to the author of the *Tractatus*, Frege believes that the two expressions that are combined together with the sign of equality have the same meaning but not

the same sense. The assumed Fregean claim can be presented as a conditional statement: if A, then B; that is, if the two expressions are combined, their meanings and senses are accordingly related. The reader should note that we have already defined clause A: it is the “Premise 6.23 [1]” entity that is a free-floating element of our knowledge base. Therefore, our task is reduced to reconstructing clause B. At the general level, it is a conjunction of the two separate clauses. In fact, we can try to represent them as follows: “Expression (TLP 6.23 [1]) variable x”—“same meaning as”—“Expression (TLP 6.23 [1]) variable y” and “Expression (TLP 6.23 [1]) variable x”—“different sense from”—“Expression (TLP 6.23 [1]) variable y.” The shortcoming of this approach is that we add the two very specific object properties to our knowledge base. If we want to avoid this, we should instead take a different approach. First, we create four additional concept variables: two for “Meaning” and the other two for “Sense” concepts. Subsequently, we declare that the “Meaning” variables are actually the same entity, while the “Sense” variables are distinct. Analogously to the case of the “Expression” variables, we can establish relationships between the four variables outside of the context of any perspective. Thus, the final structure of clause B can be slightly simpler than the structure of the premise. This is shown in Diagram 13, together with graphs representing the relationships between the four additional variables.

Finally, we are ready to represent the first sentence of thesis 6.232. This is an example of a nested perspective: our familiar “Perspective 1” includes the claim that states that “Fregean TLP 6.232 perspective”—yet another instance of the “Perspective” class—includes a claim being the conditional statement whose premise is “Premise TLP 6.23 [1]” and conclusion is “Conclusion TLP 6.232 [1].” The latter claim, unlike all the previously discussed instances of the “Claim” class, does not have “source of claim” property because it is a part of Wittgenstein’s reconstruction of Frege’s view. The whole situation is depicted in Diagram 14.

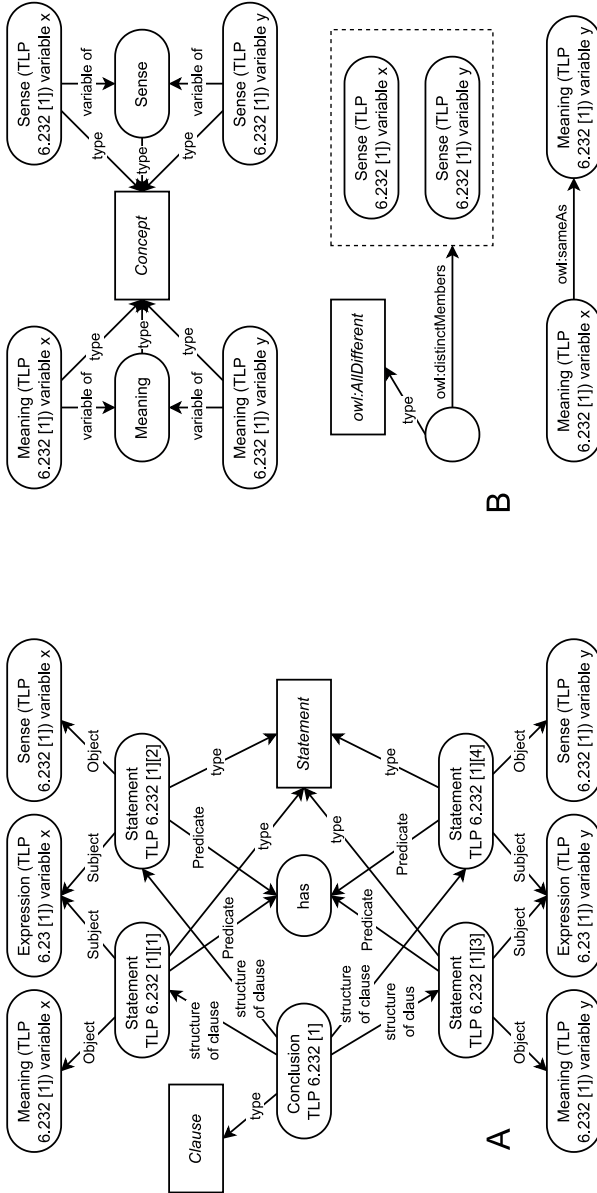


Diagram 13. Figure A is the structure of the reconstructed Fregean conclusion; figure B represents the triples that define the relationships between the four new variables. The dashed rectangle represents an RDF “Collection” entity (collections store lists of elements); the “owl:AllDifferent” class together with the “owl:distinctMembers” property make the OWL syntax that allows for the declaration of a group of individuals as distinct resources. The property “owl:sameAs” means that the related entities are actually the same resource (cf. Dean et al. 2009)

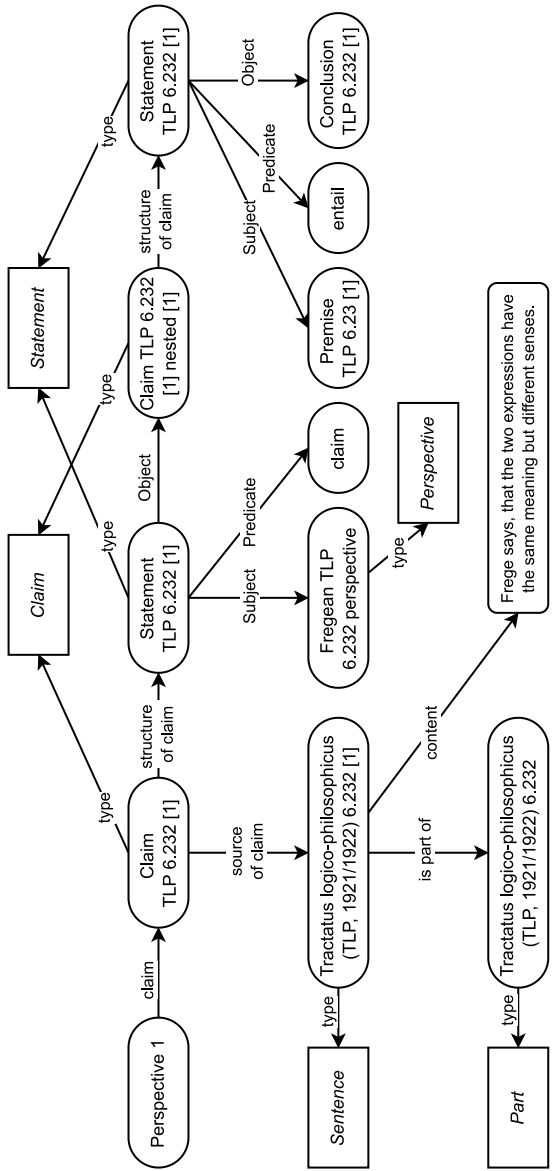


Diagram 14. The graph that represents the first sentence of thesis 6.232. The nodes “Perspective 1,” “Premise TLP 6.23 [1],” and “Conclusion TLP 6.232 [1]” are assumed to have been defined earlier

Concluding Remarks

Developing a representation of philosophical content according to the SW paradigm is a strange form of translation: we try to render a philosopher's natural language into a controlled vocabulary; that vocabulary is a result of our own choices. Therefore, it is a game we play against ourselves: we try to limit our means of expression without reducing the meaning that is conveyed. In the course of this game, we have to make a number of philosophically weighty decisions. Some of the interpretations I offer can prove controversial, such as a particular reading of the copula or an introduction of a conditional statement that is not visible in the source material. One of the reasons for these moves is that my attempt concerns the conceptual structure of the philosophical doctrines presented in Wittgenstein's published and unpublished works, and that conceptual structure can be, from time to time, hidden under the surface of its linguistic expression.¹⁰ The other is that the SW technology is generally based on binary relations, and any more complicated syntax must be represented as a structure made of them. The third reason is that I think of the *Tractatus* representation as a device that should perform a certain function once completed: it should be possible to create a user-friendly web interface that would enable scholars or students to search through the knowledge base and find useful information about Wittgenstein's conceptions. The working part of the Wittgenstein ontology project is already such a device with a comprehensive web interface. Although it does not do much more than linking dates and persons to Wittgenstein's particular remarks, its functionality enables one to check all the places in the *Nachlass* where Sigmund Freud, for example, is mentioned. The simplicity of its interface results from the fact that the Wittgenstein ontology in its current shape makes use of just a handful of properties. Unfortunately, I will not avoid a large number of object properties when I complete the "Subject" branch in a way suggested by my examples. However, I can try to reduce that number to make the project's interface design less challenging.

The SW technology was not created to represent philosophical ideas; if we try to do this anyway, we should expect our representations to be complex. Fortunately, the complexity develops according to certain patterns; therefore,

¹⁰ Such a claim does not need to presuppose a doctrine of the inner language of thought, although such a doctrine is actually proposed by the *Tractatus* itself.

they can be easily automated with the help of a piece of uncomplicated custom-made software.

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Gramatyka nieskończoności. Ludwiga Wittgensteina krytyka teorii mnogości*

Słowa kluczowe: Cantor, Dedekind, język, teoria mnogości, nieskończoność, dowód diagonalny

Keywords: Cantor, Dedekind, language, set theory, infinity, diagonal proof

Grammar of Infinity. Wittgenstein's Critique of Set Theory

Abstract

The paper discusses a relatively underexamined element of Wittgenstein's philosophy of mathematics associated with his critique of set theory. I outline Wittgenstein's objections to the theories of Dedekind and Cantor, including the confounding of extension and intension, the faulty definition of the infinite set as infinite extension and the critique of Cantor's diagonal proof. One of Wittgenstein's major objections to set theory was that the concept of the size of infinite sets, which Cantor expressed by means of symbols \aleph_0 and \mathfrak{c} , had no application, i.e., that there was no grammatical technique that could show how such expressions were to be used. Notions of set theory are, so to speak, exterior – they find themselves outdoors, outside of what we usually do. They form a discourse that takes us beyond the horizon of everydayness and commonality. They are like an engine idling of the language of mathematics.

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Wprowadzenie

Po powrocie do Cambridge w roku 1929, który był zarazem powrotem do filozofii, Wittgenstein intensywnie zajmował się filozofią matematyki, do czego z pewnością przyczyniły się dyskusje z Frankiem Ramseyem, które rozszerzyły też intelektualny horyzont omawianych problemów. Podczas gdy w okresie *Traktatu* punktem odniesienia dla filozofii logiki i matematyki Wittgensteina pozostawał logicyzm Russella i Fregego, to w okresie przejściowym w jego notatkach pojawiają się takie nazwiska, jak Skolem, Hilbert, Weyl, Brouwer, Dedekind czy Cantor. W lekturze tych notatek uderzać może jednoznaczna, nieprzejednana i bardzo ostra krytyka poglądów dwóch ostatnich z wymienionych matematyków i w ogóle całej teorii mnogości. Wittgenstein pisze o niej, że jest „fałszywa” (*falsch*) (MS 106, s. 155; PR § 145), że „matematyka jest całkowicie zanieczyszczona przez szkodliwe (*perniciöse*) wyrażenia teoriomnogościowe” (MS 106, s. 251), że jest po prostu „paplaniną” (*Geschwätz*) (BT, s. 747). Swojego negatywnego stanowiska nie zmienił także w powstałych w latach 1937–1944 *Uwagach o podstawach matematyki*, w których pisał między innymi: „Wierzę i mam nadzieję, że przyszłe pokolenia będą się śmiać z tego hokus-pokus” (UPM II, s. 22), albo: „Wyobraź sobie, że teorię mnogości wynalazł satyryk jako rodzaj parodii matematyki. – Później jednak dostrzeżono w niej pewien sens i włączono ją do matematyki. (Skoro bowiem ktoś może ją uważać za raj matematyków, dlaczego ktoś inny nie miałby jej uważać za żart?)” (UPM V, s. 7). W ostatniej wypowiedzi o teorii mnogości z *Uwag* uznał ją w końcu za coś, co „przypomina nowotwór, bez celu i sensu wyrastający z normalnego ciała” (UPM VII, s. 11).

Zaskakuje kategoriyczny ton krytyki, a przede wszystkim czas, w którym została przeprowadzona, czyli lata 30. i 40. ubiegłego wieku, kiedy teoria mnogości była już dobrze ugruntowaną dziedziną matematyki. W ogóle pierwsze ćwierćwiecze XX wieku było okresem tryumfu metod i pojęć teorii mnogości, a niektórzy widzieli w niej przewrót w matematyce nieznaną od czasów Newtona i Leibniza¹.

Cel tego artykułu jest historyczno-filozoficzny. Chciałbym odpowiedzieć na pytanie o teoretyczny kontekst i powody tak ostrej reakcji Wittgensteina na teorię mnogości w okresie średnim i późnym. Czy była po prostu

¹ Por. Kuratowski, Mostowski, 1978, s. 16.

konsekwencją jego filozofii matematyki, czy miała może jakieś inne, na przykład światopoglądowe, motywy? Żeby to uczynić, musimy przedstawić szerszy kontekst rozważań Wittgensteina dotyczących niektórych aspektów jego filozofii matematyki w postaci (1) krytyki logicyzmu w TLP, przede wszystkim teorii klas oraz (2) dyskusji z Ramseyem na temat nieskończoności. Punkty (1) i (2) stanowią prolegomena do późniejszej krytyki teorii mnogości, która będzie omówiona i oceniona w punktach (3) i (4).

1. Krytyka teorii klas w *Traktacie logiczno-filozoficznym*

W *Traktacie* Wittgenstein krytycznie odniósł się do idei sprowadzenia matematyki do logiki i odrzucił wersję logicyzmu w wydaniu Russella i Fregego. Przedstawił własne rozumienie jedności logiki i matematyki, w której nie było miejsca dla pojęcia klasy². W tezie 6.031 wyraził to wprost: „Teoria klas jest w matematyce całkiem zbyteczna. Wiąże się to z tym, że ogólność potrzebna w matematyce nie jest ogólnością *przypadkową*”. Mówiąc o „przypadkowości”, Wittgenstein miał na myśli to, że próba Russella zdefiniowania podstawowych pojęć matematyki, na przykład liczby, w terminach logicznych, a konkretnie teoriomnogościowych, oparta była na dwóch „nie-logicznych” aksjomatach: nieskończoności i redukowalności. Odrzucenie tych aksjomatów było w istocie równoznaczne z odrzuceniem teorii klas, choć oczywiście nie był to jedyny motyw. Wittgenstein w ogóle nie widział konieczności uzasadniania i uprawomocniania matematyki, tak jak nie dostrzegął takiej konieczności w logice. Wielokrotnie podkreślał, że „logika musi się sama o siebie zatroszczyć” (TLP 5.473), że teoria klas i „teoria typów”, pomyślane właśnie jako takie teorie uzasadniające, są zbyteczne.

Obiekcje wobec tej teorii klas Wittgenstein zgłaszał jeszcze przed napisaniem *Traktatu*, mianowicie w *Dziennikach 1914–1916*, w których pisał:

² Takiego zdania jest też większość badaczy, np. M. Marion stanowisko Wittgensteina w *Traktacie* nazywa „logicyzmem bez klas” i mówi, że miał on na myśli „redukcję arytmetyki do »teorii operacji«” (Marion, 1998, s. 26). Podobnie uważają P. Frascolla (1994, s. 37), M. Potter (2000, s. 177–182) oraz O. Kuusela (2019, s. 45 i nn.). Nieco inne stanowisko reprezentują: R. Rodych (1995, s. 272–274), M. Wrigley (1998, s. 129–132) oraz S. Schroeder (2020, s. 13–31), którzy zwracają z kolei uwagę na zbyt znaczące różnice w ujęciu Wittgensteina w stosunku do teorii Russella i Fregego, by można było mówić o filozofii matematyki w TLP jako pewnej wersji logicyzmu.

„Teoria klas nie uwidacznia jeszcze dostatecznie, dlaczego zdanie (*Satz*) potrzebuje swego przeciwieństwa (*Gegensatz*)” (D, s. 94). Innymi słowy, teoria ta nie ukazuje dostatecznie jasno związku między zdaniem a jego negacją. W ujęciu teorii klas negacja ($\sim p$) byłaby klasą zdań zaprzeczających p , ale w takim przypadku, gdy potraktuje się te zdania jako klasy, brakuje związku między p i $\sim p$. Wittgenstein ustanawia ten związek, wskazując na negację jako pewną operację logiczną, która dopiero wytwarza klasy, a więc jest działaniem pierwotnym wobec nich. Krótko mówiąc, Wittgenstein proponuje wyjaśnienie negacji nie w kategoriach teoriomnogościowych, ale jako operacji, w której jedno ze zdań konstruowane jest na podstawie drugiego. Myśli te rozwinął później w tezach 5.2–5.23 *Traktatu*, gdzie pojęcie operacji połączył z pojęciem stosunków wewnętrznych, określając operację jako regułę przekształcającą jedno zdanie w drugie. Po zdefiniowaniu ogólnej formy zdania, z której wynika, że każde zdanie jest rezultatem sukcesywnego stosowania operacji negacji do zdań elementarnych, oraz po podaniu ogólnej formy operacji, Wittgenstein dochodzi do definicji liczb całkowitych jako szczególnej odmiany szeregu form³, uporządkowanego przez relację wewnętrzną i powstałego w wyniku powtarzania operacji (por. TLP 4.1252; TLP 6.03).

W pojęciach operacji i szeregu form zawarte jest pojęcie indukcji zupełnej w postaci formuły „i tak dalej”, albo trzech kropek postawionych na końcu operacji logicznych i określających możliwość nieskończonego stosowania tych operacji do własnych rezultatów. Indukcja pozwala nie tylko na generowanie nieskończonych szeregów form, ale także na ustanowienie określonych związków między ich członami. Zapewnia tym samym „nieprzypadkową ogólność”, by nawiązać do tezy 6.031.

W pojęciach indukcji i operacji zawarte jest z kolei pojęcie nieskończoności, które w zasadzie jest równoważne z indukcją. Wszystkie szeregi form mają tę własność, że mogą być konstruowane w nieskończoność. Ale w jakim znaczeniu „nieskończoności”? Czy Wittgenstein miał w *Traktacie* na myśli nieskończoność aktualną czy potencjalną? Odpowiedź na to pytanie nie jest jednoznaczna. Na przykład komentując aksjomat nieskończoności Russella, napisał w tezie 5.535: „To, co ma mówić aksjomat nieskończoności, wyrażałoby się językowo przez istnienie nieskończenie wielu nazw

³ Definicję szeregu form Wittgenstein oparł na pojęciu szeregu wprowadzonym przez Fregego. Por. Frege, 1993, §29.

o różnych znaczeniach”. Nieskończoność nazw implikuje zatem nieskończoność przedmiotów. W tezie 4.2211 stwierdza natomiast: „Gdyby nawet świat był nieskończenie złożony, tak że każdy fakt składałby się z nieskończenie wielu stanów rzeczy, a każdy stan rzeczy z nieskończenie wielu przedmiotów, to i wtedy musiałyby istnieć przedmioty i stany rzeczy”. Tezy te świadczyć mogą o tym, że Wittgenstein dopuszczał w *Traktacie* możliwość istnienia nieskończoności aktualnej. Taka możliwość tkwi także w pojęciu „przestrzeni logicznej”, rozumianej jako nieskończona całość wszystkich możliwych zdań. Ponieważ poszczególne zdanie wyznacza tylko pewne miejsce w przestrzeni logicznej, przestrzeń ta musi być już dana (por. TLP 3.42) jako całość nieskończona i określona⁴.

Z powyższymi tezami, które zdają się przemawiać za nieskończonością aktualną, kontrastuje teza 4.1272, w której Wittgenstein stwierdza: „Nie można np. powiedzieć »są przedmioty« jakby się mówiło »są książki«; ani »jest 100 przedmiotów«, albo »jest \aleph_0 przedmiotów«. Nedorzecznością jest też mówić o *liczbie wszystkich przedmiotów*”. A więc zdanie „istnieje nieskończenie wiele przedmiotów” byłoby w myśl *Traktatowej* koncepcji pojęć formalnych niby-zdaniem. Wittgenstein wprowadził termin „pojęcia formalne” właśnie po to, by ujawnić źródło mieszania tych pojęć z pojęciami właściwymi (materialnymi). Pojęcia formalne, takie pojęcia jak „fakt”, „funkcja”, „zdanie”, „liczba”, „przedmiot” etc., nie dają się, jego zdaniem, poprawnie wyrazić w języku ani za pomocą pojęcia funkcji, jak chciał Frege, ani za pomocą pojęcia klasy, jak proponował Russell. W koncepcji Wittgensteina pojęcia formalne reprezentuje zmienna zdaniowa, a jej wartość przedmioty podpadające pod te pojęcia. Każda zmienna oznacza pewną stałą formę przysługującą wszystkim jej wartościom, można ją zatem traktować jako własność formalną. Jeśli pojęć formalnych używamy nie jako zmiennych zdaniowych, ale jako pojęć właściwych, wówczas powstają niedorzecznie niby-zdania. Innymi słowy, dla Wittgensteina z zakresu *Traktatu* adekwatnym wyrazem nieskończoności pozostaje „itd.” albo „...”⁵, a zatem nieskończoność można tylko „zobaczyć” w tych symbolach, wyrazić przez pewien schemat czy też wzór lub matrycę, którą powielamy przez sukcesywne jej stosowanie do kolejnych wyników operacji.

⁴ Por. Marion, 1998, s. 34.

⁵ Podkreśla to m.in. Rotter, 2006, s. 69.

2. Dyskusje z Ramseyem o nieskończoności i kwantyfikatorach

Uznanie teorii klas za zbędną w matematyce, a w logice za co najmniej drugorzędną, i zastąpienie jej pojęciami operacji oraz szeregu form, stanowiło niewątpliwie prolegomena do krytyki teorii mnogości przez Wittgensteina w okresie przejściowym. Szeroko drzwi dla tej krytyki otworzyły jednak dopiero dyskusje z Frankiem Ramseyem z roku 1929 dotyczące nieskończoności i problemu kwantyfikacji⁶. Skierowały one Wittgensteina w stronę finityzmu, który był głównym motywem jego polemiki z teoriami Cantora i Dedekinda⁷.

W *Traktacie* Wittgenstein nie widział większego problemu, by idąc za Russellem i Fregem, zdefiniować ogólny kwantyfikator jako iloczyn logiczny: $\forall x.f(x) = fa \wedge fb \wedge fc \dots$; zaś kwantyfikator szczegółowy jako sumę logiczną: $\exists x.fx = fa \vee fb \vee fc \dots$ (TLP 5.521). Rolą kwantyfikatorów było zatem wytwarzanie zdań: koniunkcji albo alternatywy. Dla Wittgensteina nie miało wówczas większego znaczenia, czy liczba członów koniunkcji albo alternatywy jest skończona czy nie. Dyskusje z Ramseyem przyczyniły się do zmiany stanowiska. Ramsey w artykule *General Proposition and Causality* wykazywał, że tylko kwantyfikacje po dziedzinach skończonych można uważać za zdania, *sc.* koniunkcje albo alternatywy⁸. Kwantyfikacje po dziedzinach nieskończonych, które nazywał zmiennymi wyrażeniami hipotetycznymi (*variable hypotheticals*), są natomiast regułami tworzenia zdań, a nie zdaniami⁹. Jeśli $\forall x.f(x)$ zastępuje nieskończony iloczyn logiczny, to znając znaczenie symbolu „ \wedge ” i zakładając, że iloczyn ten jest prawdziwy, możemy wywnioskować każde zdanie o postaci fa . W przypadku nieskończonej koniunkcji tego jednak wiedzieć nie możemy, nasze wnioskowanie

⁶ Wittgenstein i Ramsey dyskutowali problem nieskończoności i kwantyfikacji w pierwszym półroczu 1929. Dowodem tego może być dokument napisany w języku niemieckim i znajdujący się w spuściźnie piśmienniczej Ramseya, który prawdopodobnie zawierał uwagi do wykładu o nieskończoności w matematyce. Wittgenstein wygłosił go na konferencji połączonych sekcji Towarzystwa Arystotelicznego i Badań nad Umysłem w Nottingham, zamiast wykładu *Kilka uwag o formie logicznej*, który uznał za niedobry. Niestety tekst wykładu o nieskończoności nie zachował się. Więcej na ten temat Kienzler, 1997, s. 56–62; Mathven, 2020.

⁷ Por. Rodych, 2000, s. 285.

⁸ Por. Ramsey, 1950, s. 237–244.

⁹ Tamże, s. 241 i nn.

jest więc wysoce niepewne, a co ważniejsze, nierozstrzygalne. Nie wiemy, czy gdzieś nie spotkamy „ $\sim fa$ ”, które jest niezgodne z $\forall x f(x)$, gdyż wyrażenie to oznacza, że funkcja zdaniowa fx jest prawdziwa we wszystkich przypadkach. W konsekwencji nie moglibyśmy niezawodnie wnioskować z dowodu $\sim \forall x \sim f(x)$, że $(\exists x) fx$, na co zwracał szczególną uwagę David Hilbert¹⁰.

Wittgenstein na przełomie lat 20. i 30. doszedł do wniosku, pod wpływem dyskusji z Ramseyem¹¹, że ogólny kwantyfikator możemy zapisać jako koniunkcję $fa \wedge fb \wedge fc \dots$, tylko jeśli trzy kropki „ \dots ” potraktujemy jak tak zwane „kropki z lenistwa” (*the dots of laziness*)¹², stawiane w miejsce skończonej ilości zdań będących argumentami funkcji prawdziwościowej. Analogicznie jak przy wymienianiu liter alfabetu: a, b, c, ..., ale nie na przykład przy zapisywaniu ciągu liczb naturalnych 1, 2, 3... Tylko wtedy, kiedy koniunkcja jest skończona, czyli kiedy „ \dots ” są „kropkami z lenistwa”, możemy skonstruować sensowne zdanie zastępujące $fa \wedge fb \wedge fc \dots$. Jeżeli zaś zmienna nie jest ograniczona do dziedziny skończonej i może przyjmować nieskończenie wiele wartości, tego zrobić się nie da. Mając ponadto na uwadze *Traktatowy* postulat określoności sensu (TLP 4.023) oraz to, że w przypadku nieskończonej długiej koniunkcji uchwycenie warunków prawdziwości każdego z jej członów nie jest z zasady możliwe, zdanie takie jest pozbawione sensu. Zdanie ma bowiem sens, kiedy wiemy, w jakich okolicznościach jest prawdziwe, a w jakich fałszywe. Wiedzieć

¹⁰ Por. na ten temat Methven, 2015, s. 201–202.

¹¹ W literaturze przedmiotu istnieją spory o to, kto inspirował kogo. Niestety sporów tych nie możemy tu omawiać (zrobił to u nas J. Gomułka, 2016, s. 145–157). Zdaniem Methvena (por. Methven, 2020, s. 1108–1133) Wittgenstein i Ramsey różnili się w ujęciu sposobu, w jaki nieskończoność pojawia się w naszym myśleniu, ale obaj byli zgodni, że nie pojawia się ona jako nieskończoność aktualna. Methven stanowisko Ramsey'a określił jako radykalnie finitystyczne, Wittgensteinowi natomiast przypisał *holistyczne* rozumienie nieskończoności. Polega ono z grubsza na tym, że nieskończoność jest *implicite* zawarta w naszej gramatyce pewnych dyskursów dotyczących przestrzeni, czasu i kolorów. Moje pojęcie, np. nieskończonej przestrzeni, nie jest budowane niezależnie od mojego doświadczenia skończonej przestrzeni, ponieważ musi ono obejmować to, co nieskończone. Zatem to, co nieskończone, musi już być jakoś obecne w moim myśleniu, ale oczywiście nie jako niezależne od mojego doświadczenia, raczej jako zawarte w nim *implicite* – przynajmniej w niektórych obszarach. Interpretacja Methve'a wymagałaby jednak oddzielnej opracowania.

¹² Sformułowanie to znamy z notatek Moore'a z wykładów Wittgensteina. Por. WLC, s. 219.

to możemy tylko wtedy, gdy zmienna przyjmuje skończoną ilość wartości (z tego względu także nieskończona suma logiczna jest niedorzecznością). Dlatego też, by uczynić sensownym zdanie $\forall x.f(x) = fa \wedge fb \wedge fc \dots$, musimy z nieskończonego iloczynu logicznego zrobić skończony: $\forall x.f(x) = fa \wedge fb \wedge fc \dots fn$. To jednak jest możliwe tylko, jeśli przyjmiemy, jak zauważa Glock¹³, „aksjomat skończoności” mówiący, że liczba przedmiotów w świecie jest skończona. Problem w tym, że liczba przedmiotów w świecie nie jest kwestią logiczną, tylko empiryczną. Aksjomat taki byłby więc nie-logiczną podstawą logiki, którego prawdziwość i konieczność jest fundamentalnie wątpliwa, tak jak „aksjomat nieskończoności” w Russellowskiej „teorii typów”, który Wittgenstein zdecydowanie krytykował i odrzucał.

3. Między rachunkiem a prozą

Dyskusje z Ramseyem uświadomiły Wittgensteinowi, jak trudno mówić o nieskończoności i że być może unikniemy w logice i matematyce związanych z nią trudności, jeśli ograniczymy się do dziedzin skończonych. Taka postawa nie dziwiła, zważywszy, że ówczesna matematyka była w większości finitystyczna. Pojawienie się teorii mnogości Cantora, którą Hilbert przyrównał do matematycznego raję, potraktowano więc jako znaczące wydarzenie. Tego optymizmu nie podzielał Wittgenstein, widząc w teorii mnogości raczej piekło¹⁴. Ten pierwszy był matematykiem, ten drugi filozofem. Precyzyjniej rzecz ujmując, moglibyśmy powiedzieć, że wypowiedzi autora *Traktatu* na temat teorii stworzonej przez Cantora nie są wypowiedziami matematyka, lecz raczej filozofa matematyki. Z pewnością byłaby to prawda, i to raczej niekontrowersyjna. Co więcej, Wittgenstein dążył do ścisłego odróżnienia tego, co w matematyce jest rachunkiem (*sc.* algorytmem), od wszystkiego, co nim nie jest. Pisał na przykład:

Jest osobliwym błędem matematyków, że wielu z nich sądzi, iż przez krytykę podstaw coś z matematyki może odpaść. Część matematyków ma całkiem dobry instynkt: to, co już raz *policzyliśmy*, nie może wypaść i zniknąć. To, co rzeczywiście znika za sprawą krytyki, to nazwy, aluzje, które występują w rachunku, a więc to, co nazwałbym *prozą*.

¹³ Por. Glock, 2001, s. 236.

¹⁴ Por. UPM V, s. 7; LFM, 103.

Bardzo ważne jest ściśle rozróżnienie między rachunkiem a prozą. Skoro już raz dokona się wyraźnie tego podziału, odpadną wszystkie te pytania o niesprzeczność, niezależność etc.

(WWK, s. 149)

Przez „prozę” autor tych słów rozumiał zarówno filozoficzną interpretację rachunków i dowodów, jak i stwierdzenia typu: „Istnieje nieskończenie wiele liczb pierwszych” czy też: „Zbiór liczb rzeczywistych jest większy od zbioru liczb wymiernych”¹⁵. Przy czym „proza” nie jest u Wittgensteina przeciwstawiona prawdzie jako coś, co jest fałszywe samo w sobie, choć niektóre jego wypowiedzi, jak powyższa, mogłyby to sugerować. Należały raczej odróżnić prozę błędną od prozy rozjaśniającej, jakkolwiek to odróżnienie w dyskursie Wittgensteina wydaje się niekiedy trudne do przeprowadzenia, jeśli wziąć po uwagę na przykład jego zdecydowaną krytykę metody przekątniowej Cantora czy przekrojów Dedekinda i jego definicji zbioru nieskończonego. Ta krytyka uprawnia do pytania: czy Wittgenstein neguje nie tylko błędną prozatorską interpretację teorii mnogości, ale także jej sens matematyczny (rachunkowy)?

Hilary Putnam odpowiedział na to pytanie twierdząco¹⁶. Jego zdaniem wbrew temu, co Wittgenstein wielokrotnie podkreślał, mianowicie że w filozofii nie wolno wysuwać żadnych tez, sam je w kontekście krytyki teorii mnogości wysuwa. I są to wedle Putnama tezy całkowicie błędne, a jego filozoficzne badania dotyczące niezmiernie ważnych kwestii pojęciowych wymagały o wiele większej wiedzy naukowej i o wiele więcej respektu dla nauki. W następnych paragrafach postaramy się rozstrzygnąć, czy ocena Putnama jest słuszna, czy Wittgenstein odrzuca teorię mnogości jako błędną, czy jedynie kwestionuje jej opaczną interpretację filozoficzną.

3.1. Pomieszanie kontekstów intensjonalnych z ekstensjonalnymi

Uświadomienie sobie problemów, które rodziła kwantyfikacja po dziedzinach nieskończonych, skłoniło Wittgensteina do baczniejszego przyjrzenia się znaczeniom słów „skończony” i „nieskończony” oraz kategorialnej różnicy między nimi. W jednej z pierwszych notatek, sporządzonych zaraz

¹⁵ Więcej na temat pojęcia „prozy” u Wittgensteina zob. Shanker, 1987, s. 161–198.

¹⁶ Por. Putnam, 2007, s. 236, 246.

po powrocie do Cambridge, Wittgenstein odwołał się do dyskusji z Ramseyem o nieskończoności:

Powiedziałem ongiś, że ekstensjonalna nieskończoność nie istnieje. Ramsey odpowiedział na to, że można sobie wyobrazić człowieka, który żył wiecznie, tzn. po prostu nie umarł. Czy nie jest to nieskończoność ekstensjonalna? Na pewno mogę sobie wyobrazić, że koło obraca się i *nigdy* nie zatrzymuje. Tkwi tu pewna osobliwa trudność: wydaje mi się czymś poniekąd przypadkowym (niedorzeczne powiedzenie), że w pokoju jest nieskończenie wiele przedmiotów. Natomiast mogę sobie pomyśleć *intensjonalnie* NIESKOŃCZONE PRAWO (albo *nieskończoną regułę*), na mocy której wytwarza się ciągle coś nowego – *ad infinitum* – ale naturalnie tylko to, co może wytworzyć reguła, czyli konstrukcje.

Zatem wydaje się, że nieskończone obroty koła mogą być konstrukcjami, podczas gdy nowych przedmiotów konstruować nie mogę. (...) Mam wrażenie, że sam tylko negatywny opis tego, co *nie ma końca*, nie może dostarczać nieskończoności pozytywnej.

(MS 105, s. 23–27)

Możemy więc przyjąć, że już na początku roku 1929 Wittgenstein doszedł do wniosku, że istnieje zasadnicza różnica między ekstensjonalnym a intensjonalnym punktem widzenia na nieskończoność. Tej różnicy nie dostrzegał wyraźnie w *Traktacie*, często przechodząc od jednego kontekstu do drugiego, podobnie jak Cantor, Dedekind czy Frege¹⁷. Wittgenstein stanął jednak na stanowisku, że istnieje zasadnicza różnica między zbiorem utworzonym w sposób ekstensjonalny i intensjonalny. Ten pierwszy sposób polega po prostu na wylczeniu elementów zbioru, podaniu listy jego elementów. W *The Big Typescript* skonstatował: „Symbolem dla klasy jest lista” (BT, s. 740). Drugi natomiast na wskazaniu reguły generowania poszczególnych elementów zbioru czy też przytoczenia ogólnej charakterystyki jego elementów.

¹⁷ We współczesnej teorii mnogości nie stosuje się tego podziału. Zdaniem F. Mühlhölzera (Floyd, Mühlhölzer, 2020, s. 30) termin „intensjonalny” nie jest adekwatny, ponieważ sugeruje, że znaczenie słowa to pewien byt (*entity*) zwany „intensją”, a taki pogląd Wittgenstein odrzucał. Należy jednak zauważyć, że sam Wittgensteina używał tego terminu i jasno go definiował jako prawo bądź regułę, na mocy której konstruuje się pewne formy, a nie jako mentalny byt zwany znaczeniem.

Zdaniem Wittgensteina jednym z błędów teorii mnogości jest twierdzenie, że można zrozumieć znaczenie słowa „zbiór” nie wiedząc, czy jest on skończony czy nieskończony, że to możemy ustalić dopiero później¹⁸. Tak jednak nie jest, ponieważ w obu przypadkach słowo „zbiór” znaczy coś zupełnie innego. Różnica między zbiorami skończonymi i nieskończonymi jest różnicą kategoryalną – tego, co możemy sensownie powiedzieć o pierwszych, nie da się sensownie powiedzieć o drugich. Na przykład skończony zbiór wszystkich liczb parzystych mniejszych od 10 możemy po prostu wyliczyć, podając kolejne jego elementy, sporządzić ich listę czy też wskazać jego ekstensję. W przypadku zbiorów nieskończonych podobnej listy stworzyć nie możemy. Zbiór nieskończony daje się wygenerować tylko intensjonalnie. Wittgenstein twierdził zarazem, że poprawny symbolizm powinien przedstawiać zbiór nieskończony inaczej niż skończony. Niestety nie przedstawił żadnej propozycji takiego nowego symbolizmu. Podkreślał natomiast z naciskiem, że „*Nieskończoność nie jest liczbą*. Słowo »nieskończony« ma inną składnię niż liczebnik” (WWK, s. 228). „Nieskończoność” nie oznacza też tyle, co ogromnie dużo, jak w zdaniu, że „istnieje nieskończenie wiele gwiazd”, jakby słowo to przedstawiało rzeczywistość. „W sposób fałszywy operuje się słowem »nieskończony«, jak liczebnikiem; ponieważ jedno i drugie w języku potocznym odpowiada na pytanie *ile...*” (BT, s. 742). W notatkach zauważa natomiast: „Nieskończoność nie jest wielkością, ale wygląda jak wielkość. To jest nasza trudność” (MS 111, s. 190).

Wittgenstein rozumiał nieskończoność potencjalnie i kontynuował w tym tradycję sięgającą jeszcze Arystotelesa¹⁹. Nieskończoność jest możliwością nieskończonego stosowania reguły generującej poszczególne elementy danego ciągu albo zbioru. Przy czym należy podkreślić, co jest bardzo ważne, że tę nieskończoną możliwość stosowania reguły ujmował negatywnie jako działanie, które nie ma końca²⁰: „Nieskończony ciąg liczb jest jedynie nieskończoną możliwością skończonych ciągów liczb. Bezsensowne jest mówienie o *całych* nieskończonych ciągach liczbowych, jak gdyby były także ekstensją” (PR, § 144). W tym sensie Wittgenstein odrzucał nieskończoność

¹⁸ Por. WWK, s. 228.

¹⁹ Więcej na temat tej tradycji por. Moore, 1991, s. 206–208.

²⁰ Por. PR, § 139. Dla Putmana (2007, s. 240) takie negatywne ujęcie nieskończoności w matematyce jest całkowicie nieadekwatne. Jego zdaniem nawet w podejściu intuicjonistycznym obecne jest twierdzenie pozytywne, że zawsze istnieje możliwość kontynuacji danego ciągu. Jest aksjomatem arytmetyki, że każda liczba ma następnik.

aktualną we wszelkich jej manifestacjach. Nieskończoność pozostawała dla niego własnością naszego języka, nie rzeczywistości, własnością prawa, a nie jego ekstensji. Moglibyśmy takie ujęcie nieskończoności nazwać gramatycznym i przeciwstawić je ujęciu ontologicznemu Cantora²¹, dla którego nieskończoność potencjalna jest pewną nieokreśloną zmienną wielkością skończoną, rosnącą albo malejącą poza wszystkie skończone granice. Jako taka nie jest w istocie żadną nieskończonością, dlatego w *Grundlagen einer allgemeinen Mannigfältigkeitslehre* nazywał ją nieskończonością niewłaściwą²². Nieskończoności potencjalnej, którą określał także jako „przymilną iluzję”, Cantor przeciwstawiał nieskończoność aktualną, która jest stała we wszystkich swych częściach i zarazem przekracza każdą wielkość skończoną tego samego rodzaju. Przykładem może tu być ogół wszystkich skończonych liczb całkowitych dodatnich, tworzący pewien zbiór jako określoną wielkość, rzecz dla siebie (*ein Ding für sich*), niezależnie od ciągu naturalnego należących do niego liczb. Nieskończoność potencjalna ciągle wskazuje na nieskończoność aktualną, która jest jej warunkiem i użycza jej realności. Jeśli bowiem definiujemy nieskończoność jako nieskończone stosowanie reguły, to taką nieskończoność aktualną musimy już założyć jako warunek możliwości tego nieskończonego stosowania.

Wittgenstein rzecz widział całkiem inaczej. Tak jak nie możemy o tym, co możliwe, mówić w ten sam sposób, jak o tym, co rzeczywiste, tak też istnieje, jego zdaniem, zasadnicza różnica kategoryalna w przypadku zbiorów skończonych i nieskończonych. Niestety tej różnicy kategoryalnej nie dostrzega teoria mnogości. Na przykład Cantor przyjmował, że moc zbioru nieskończonego jest pod względem kategoryalnym tym samym, co moc

²¹ Por. Cantor, 2003, s. 185.

²² J. Ferreirós (2007, s. 18–19) zwraca uwagę na filozoficzną atmosferę w Niemczech XIX w., która sprzyjała mówieniu o nieskończoności aktualnej. Hegel na przykład określał nieskończoność potencjalną jako „złą nieskończoność” i przeciwstawiał ją *nieskończoności jakościowej*, właściwej Absolutowi. Cantor z kolei w jednej ze swych prac przytoczył słowa Leibniza: „Tak bardzo optuję za nieskończonością aktualną, że zamiast przyznawać, że przyroda jej nie znosi, jak to się potocznie mówi, twierdzę, że działa ona w niej wszędzie po to, by lepiej zaznaczyć ślad doskonałości jej Twórcy. Tak więc sadzę, że nie istnieje żadna część materii, o której nie powiem, że jest podzielna, ale że jest aktualnie podzielona; w konsekwencji najmniejsza cząstka materii musi być rozpatrywana jako świat pełen nieskończoności rozmaitych stworzeń” (Cantor, 1932, s. 179). Na temat związków Cantora z teologami i filozofami zob. także Murawski, 2012, s. 211–237; Murawski, 2018, s. 221–242.

zbioru skończonego. Co więcej, w ocenie Wittgensteina teoria mnogości nie tylko nie odróżnia kontekstów ekstensjonalnych i intensjonalnych, ale miesza je ze sobą, powołując do życia obiekty, *sc.* „zbiory nieskończone”, „liczby nieskończone” jako „nieskończone całości”, które po prostu nie istnieją.

Jak już wspominaliśmy, pojęcie „zbioru” było dla Wittgensteina symbolem oznaczającym listę (ekstensję), ale w istocie zbiór tworzymy w sposób intensjonalny, to znaczy na mocy reguły. Teoria mnogości przedstawia więc zbiór nieskończony jako ekstensję, podczas gdy w rzeczywistości zbiór taki jest pojęciem intensjonalnym, to znaczy jest interpretowany jako *reguła* generowania ekstensji.

3.2. Krytyka teorii mnogości w *The Big Typescript*

Najdłuższym tekstem średniego Wittgensteina na temat teorii mnogości jest 138. rozdział *The Big Typescript* (1933) zatytułowany „Teoria mnogości”. W zasadzie jest on w całości poświęcony teorii Dedekinda, jego definicji zbioru nieskończonego i konstrukcji liczb rzeczywistych. Rozważania kontynuowane są w rozdziałach 138–140. Jeśli chodzi o pierwszą z wymienionych kwestii, to przypomnijmy, że Dedekind definiował zbiór nieskończony w następujący sposób: zbiór jest nieskończony wtedy, gdy jest równoliczny z jakimś swoim podzbiorem właściwym.

W interpretacji Wittgensteina definicja zbioru nieskończonego Dedekinda głosi, że z powodzenia bądź niepowodzenia próby przyporządkowania właściwej podklasy całej klasie ma wynikać, czy jest ona nieskończona czy nie. Zdaniem Wittgensteina jednak, taka rozstrzygająca próba nie istnieje, nie możemy efektywnie rozstrzygnąć, czy zbiór jest równoliczny ze swoim podzbiorem. „Klasa nieskończona” i „klasa skończona” są różnymi kategoriami logicznymi. Dla klasy skończonej zdanie, że nie jest ona równa swojej podklasie, jest po prostu tautologią.

To, co nazywamy „przyporządkowaniem wszystkich elementów innej klasie” w przypadku klasy skończonej, jest zupełnie czymś innym niż to, co nazywamy np. przyporządkowaniem wszystkich liczb kardynalnych wszystkim liczbom wymiernym. Oba przyporządkowania, albo to, co oznacza się tym słowem, należą w tych dwóch przypadkach do dwóch różnych typów logicznych. „Klasa nieskończona” nie jest klasą, która zawiera więcej elementów niż klasa skończona, w potocznym sensie słowa „więcej”. Jeśli mówimy, że liczba nieskończona jest

większa od liczby skończonej, to nie robi to obu liczb porównywalnymi, ponieważ w tej wypowiedzi słowo „większa” ma *inne znaczenie* niż np. w zdaniu „5 jest większe od 4”.

(BT, s. 743)

Drugim obiektem ataku Wittgensteina była konstrukcja liczb niewymiernych przez Dedekinda, który definiował liczbę niewymierną jako przekrój liczb wymiernych niewyznaczony przez żadną liczbę wymierną (tzw. przekroje Dedekinda). Swoją teorię oparł na pewnej intuicji nazwanej „zasadą ciągłości”, która głosi: „Jeżeli wszystkie punkty prostej rozpadają się na dwie klasy tego typu, że każdy punkt pierwszej klasy leży na lewo od każdego punktu drugiej, to istnieje jeden i tylko jeden punkt, który tworzy ten podział wszystkich punktów na dwie klasy, czyli rozcięcie prostej” (Dedekind, 2003, s. 158). Dedekind rozwinął swoją konstrukcję liczb niewymiernych właśnie na podstawie tej zasady. Przekrojem nazwa on podział wszystkich liczb wymiernych \mathbb{Q} na dwie klasy A, B takie, że każda liczba a z pierwszej klasy A jest mniejsza od każdej liczby b z drugiej klasy B . Symbolicznie:

1. $A \neq \emptyset, B \neq \emptyset$
2. $A \cup B = \mathbb{Q}$
3. $\forall a \in A, b \in B: a < b$

Każda liczba wymierna wytwarza przekrój, jeśli w A istnieje liczba największa albo w B istnieje liczba najmniejsza. Zdaniem Dedekinda łatwo przekonać się jednak, że istnieje nieskończenie wiele przekrojów, które nie są wytwarzane przez żadną liczbę wymierną. Jest tak w sytuacji, kiedy w A nie ma liczby największej, a w B nie ma liczby najmniejszej. Jako przykład podaje przekrój wyznaczony za pomocą liczby naturalnej D niebędącej kwadratem żadnej liczby całkowitej. Jeśli do klasy B przyjmujemy każdą dodatnią liczbę wymierną b , której kwadrat jest $> D$, natomiast do klasy A wszystkie pozostałe liczby wymierne a , to podział ten tworzy przekrój, to znaczy każda liczba a jest mniejsza od każdej liczby b . W przekroju tym nie ma jednak ani największej liczby wymiernej w A , ani najmniejszej takiej liczby w B , a zatem przekrój wyznaczony jest przez liczbę niewymierną, można bowiem łatwo udowodnić, że nie istnieje liczba wymierna, której kwadrat $= D$. W *The Big Typescript* Wittgenstein odniósł się wprost do metody Dedekinda:

Definicja cięcia Dedekinda pretenduje do bycia czymś oglądowym, skoro powiada: istnieją trzy przypadki: albo klasa R ma pierwszy człon,

a L nie ma ostatniego etc. W istocie 2 z tych 3 przypadków nie dają się pomyśleć. Chyba że słowa „klasa”, „pierwszy człon”, „ostatni człon” zmieniają całkowicie swoje potoczne znaczenie, rzekomo zachowane. Jeśli mówimy oto – zdumieni tym, że ktoś rozprawia o klasie punktów, która leży na prawo od danego punktu i nie ma początku – daj nam przykład takiej klasy, to on wyciąga przykład liczb wymiernych! Ale tutaj nie ma żadnej klasy punktów w sensie źródłowym!

(BT, s. 739)

Punkt przecięcia się dwóch krzywych nie jest wspólnym członem dwóch klas punktów, ale przekrojem dwóch praw. Chyba że pierwsze wyrażenie definiuje się, bardzo błędnie, za sprawa drugiego.

(BT, s. 739)

Wittgenstein zarzucił teorii Dedekinda, że przekrój nie jest operacją arytmetyczną, że liczby reprezentujące przekroje nie są generowane na mocy prawa.

Jeśli według Dedekinda liczby wymierne dzielą się na dwie klasy, to jak można rzeczywiście dokonać tego podziału bez jakiegoś prawa. Nie mogąe wszak wyliczyć liczb wymiernych po każdej ze stron. Chociaż geometrycznie udaje się to łatwo.

(MS 106, s. 76)

Przekrój Dedekinda mógłby być uznany za metodę określającą liczbę rzeczywistą tylko pod warunkiem, że byłyby wyznaczone przez arytmetyczne prawo, które wyraźnie klasyfikowałoby liczby wymierne do dwóch zbiorów i w tym sensie stanowiłoby podstawę decyzji, do której z tych dwóch zbiorów należy dana liczba wymierna²³. Przekroje Dedekinda nie są, zdaniem Wittgensteina, takimi konstrukcjami, i w tym sensie nie są regulowane przez arytmetyczne prawo. Przedstawiają natomiast obraz niekończącego się procesu przybliżania, który jednak pozostawia nas nieskończenie daleko od poszukiwanej liczby rzeczywistej. Wittgenstein pyta dlatego: „Czym różni się nieskończenie skomplikowane prawo od braku prawa?” (BT, s. 767). Dla niego liczby niewymierne to nie są ekstensje, ale *reguły*, czy też prawa, tworzenia rozwinięć dziesiętnych. Te reguły nazywamy liczbami po prostu dlatego, że możemy za ich pomocą liczyć podobnie jak na liczbach

²³ Por. Da Silva, 1993, s. 93.

wymiernych²⁴. Na przykład $1/7$ jest inną regułą tworzenia rozwinięcia dziesiętnego, powiedzmy bardziej bezpośrednią, niż $\sqrt{2}$. Tymczasem przekroje Dedekinda są raczej pewnym ekstensjonalnym wyobrażeniem mającym swe źródło w geometrii.

Dochodzimy tu do trzeciego zarzutu wobec Dedekinda teorii liczb rzeczywistych jako przekrojów liczb wymiernych. Wittgenstein uznawał za błędną samą ideę, że istnieje punkt korespondujący na przykład z liczbą $\sqrt{2}$. Ta idea jest konsekwencją przyjęcia, że prosta składa się z punktów oraz luk między nimi, które zgodnie z zasadą ciągłości należałoby wypełnić i które to punkty i luki istnieją jakby uprzednio wobec naszych konstrukcji. „Matematyka jest całkowicie zanieczyszczona przez szkodliwe wyrażenia teoriomnogościowe. Przykładem tego jest mówienie, że prosta składa się z punktów”. Dla Wittgensteina linia prosta nie jest zbiorem punktów, ale „*prawem* posuwania się dalej” (UPM V, s. 36). Matematycy snują błędne analogie między liczbami rzeczywistymi i geometryczną linią złożoną z punktów, które odpowiadają tym liczbom. Dedekind, wiedziony takim obrazem, dążył do wypełniania rzekomych luk w imię niezbywalnej idei ciągłości²⁵.

Również więc w przypadku krytyki przekrojów Dedekinda głównym argumentem Wittgensteina było pomieszanie ujęcia ekstensjonalnego z intensjonalnym²⁶. Nie akceptował on myślenia o punktach i odpowiadających im liczbach. Jeśli przyjmiemy, że przecinając prostą w dowolnym punkcie, natrafimy albo na liczbę wymierną, albo niewymierną, to zakładamy, że te liczby uprzednio już istnieją, niezależnie od naszych konstrukcji. Autor *Traktatu* odrzucał zatem ekstensjonalną reprezentację liczb wymiernych i niewymiernych za pomocą prostej i punktów, uznając przekroje Dedekinda

²⁴ Wittgenstein nie był pierwszym, który postulował arytmetyczną konstrukcję liczb niewymiernych. Podobnie myślał H. Weyl w pracy *Das Kontinuum* (1928), w której odrzucał liczby niewymierne niebędące arytmetycznie zdefiniowanymi przekrojami liczb wymiernych. Da Silva zwraca uwagę na żyjącego w czasach Kanta matematyka Augusta W. Rehberga, który głosił intensjonalną teorię liczb niewymiernych. Por. Da Silva 1993, s. 95.

²⁵ Ten aspekt krytyki Dedekinda akcentuje szczególnie S. Shanker (por. 1987, s. 187 i nn.). Z kolei Putnam twierdzi, że to, iż Wittgenstein kwestionował matematyczną ciągłość, która, jego zdaniem, jest wszystkim dla współczesnej matematyki, pokazuje, że szedł on złą drogą (por. Putnam, 2007, s. 245).

²⁶ Por. Bernays, 1959, s. 19.

za działania motywowane geometrycznie²⁷. Biorąc pod uwagę finitystyczne i konstruktywistyczne stanowisko Wittgensteina w filozofii matematyki, chciałby on wprowadzić przekroje nie jako zbiory, ale prawa (*resp.* reguły) dla takich zbiorów. Chodziłoby mu zatem o coś wcześniejszego niż przekrój, o coś, co dopiero ten przekrój generuje. Jak zauważa jednak Paul Bernays²⁸, w takim przypadku albo używa się bardzo niejasnego pojęcia „prawa”, albo, na co zwrócił z kolei uwagę Weyl, wpada w błędne koło w podstawach analizy związanej z definicjami niepredykatywnymi. Zdaniem Bernaysa Dedekindowi udało się tego uniknąć, reprezentując konsekwentnie inkryminowane przez Wittgensteina podejście ekstensjonalne. Przyjął za coś intuicyjnie ważnego i niewymagającego dalszego uściślenia i redukcji pojęcie zbioru liczb naturalnych i w konsekwencji zbioru liczb wymiernych.

Krytyka Wittgensteina nie zawsze jednak oddawała należną sprawiedliwość teorii Dedekinda. W pracy *Ciągłość a liczby niewymierne* Dedekind wyraźnie stwierdzał, że odwołanie się do geometrycznych intuicji jest użyteczne z dydaktycznego punktu widzenia, ale jemu chodzi o twierdzenia „dowodzone w sposób czysto arytmetyczny” (Dedekind, 2003). Dystansując się wobec dotychczasowego sposobu wprowadzenia liczb niewymiernych jako wyniku mierzenia wielkości ciągłych za pomocą innej takiej wielkości, mówił: „zamiast tego żądam, by arytmetyka rozwijała się sama z siebie” (Dedekind, 2003, s. 157), co oznaczało między innymi, by liczby niewymierne zdefiniować w pełni za pomocą liczb wymiernych²⁹.

²⁷ Por. na ten temat Marion, 1998, s. 211. W *Uwagach o podstawach matematyki* przekroje te nazywał „zwoźniczymi ilustracjami” (UPM V, s. 29), czyli takimi, które nie są zastosowaniami analizy: „W ekstensjonalnym podejściu Dedekinda myśląca jest idea, że liczby rzeczywiste znajdują się na osi liczb. Możemy je znać lub nie; to niczego nie zmienia. Wystarczy tylko dokonać przekroju lub podziału na klasy, by wskazać im wszystkim miejsce” (UPM V, s. 37).

²⁸ Por. Bernays, 1959, s. 20.

²⁹ M. Marion zdaje się jednak przyznawać rację Wittgensteinowi, kiedy stwierdza, że pomimo deklaracyjnych intencji definicja liczb rzeczywistych jako przekroi była motywowana geometrycznie, o czym świadczyć mogą następujące wypowiedzi Dedekinda: „Rzeczą najważniejszą jest teraz fakt, że na prostej znajduje się nieskończenie wiele punktów, które nie odpowiadają żadnej liczbie wymiernej (...). Jeśli chce się teraz, a taki przecież jest nasz zamiar, zbadać arytmetycznie wszystkie zjawiska mające miejsce na prostej, to liczby wymierne nie wystarczają do tego i nieodzowne staje się ulepszenie instrumentu R, który otrzymany został poprzez stworzenie liczb wymiernych. Ulepszenia tego dokonać można poprzez stworzenie nowych liczb tego rodzaju, że dziedzina liczb

Putnam zwraca uwagę na inny jeszcze aspekt inkryminowanego przez Wittgensteina związku między punktami w przestrzeni a liczbami³⁰. Chodzi o to, że od czasu wynalezienia przez Kartezjusza geometrii analitycznej, co było zarazem początkiem matematycznej fizyki, pojęcie punktu w przestrzeni – dzisiaj czasoprzestrzeni – zależne jest od pojęcia liczby rzeczywistej. To znaczy – każdy punkt w przestrzeni może być zdefiniowany za pomocą trzech liczb rzeczywistych (trzech współrzędnych). Jeśli byłoby tak, jak twierdził Wittgenstein, mianowicie, że pojęcie liczby rzeczywistej jest nieokreślone, to równie nieokreślone byłoby pojęcie punktu w przestrzeni. Jak za chwilę pokażemy, Wittgenstein mówił, że „nie ma zbioru liczb niewymiernych”, ale w takim razie równie dobrze moglibyśmy powiedzieć: „nie ma punktów w przestrzeni”. Pamiętajmy także, że we współczesnej fizyce pojęcie cząstki straciło swą fundamentalną rolę na rzecz pojęcia pola. Wielkość pola, na przykład pola elektromagnetycznego czy grawitacyjnego, może być mierzona za pomocą liczb rzeczywistych w każdym punkcie przestrzeni. Jeśli zatem pojęcie „punktu w przestrzeni” byłoby nieokreślone tak, jak nieokreślone jest pojęcie liczby rzeczywistej, to w konsekwencji także nieokreślone byłoby pojęcie pola.

4. Metoda przekątniowa Cantora, czyli hokus-pokus

Jak wiadomo, jednym z wielkich osiągnięć Cantora było udowodnienie, że liczb rzeczywistych jest *więcej*³¹ niż liczb naturalnych, całkowitych i wymiernych, których zbiory są przeliczalne. Tak nie jest jednak w przypadku liczb rzeczywistych, czego dowodził Cantor, posługując się tak zwaną metodą przekątniową. Wittgenstein podawał w wątpliwość, czy ogólnie akceptowany

uzyska tę samą zupełność, czy, jak równoważnie chcemy to wyrazić, tę samą *ciągłość*, która przysługuje prostej” (Dedekind, 2003, s. 156–157). Według Mariona intencją Dedekinda było uzupełnienie dziedziny liczb wymiernych o liczby niewymierne i uzyskanie sposobem arytmetycznym ciągłości, jaka przysługuje prostej. Dedekind rozciągnął więc zasadę geometrycznej ciągłości na arytmetykę. Wedle Wittgensteina dowód Dedekinda, że istnieje nieskończenie wiele przekroji liczb wymiernych, jest uprawomocniony przez wyobrażenie ciągłości, podczas gdy powinno być na odwrót, tzn. to wyobrażenie powinno być uprawomocnione przez dowód. Por. Marion, 1998, s. 211.

³⁰ Por. Putnam, 2007, s. 244.

³¹ Oczywiście słowo „więcej” należy tu rozumieć w sensie pojęcia mocy zbioru.

rezultat metody przekątniowej – odkrycie pewnej substancjalnej prawdy matematycznej – jest adekwatną interpretacją tej metody.

Większość fragmentów na temat metody przekątniowej pochodzi z *Uwag o podstawach matematyki* z około 1938 roku, ale o metodzie Cantora Wittgenstein wspomina już w notatkach z roku 1930. Krytykuje on nie tyle samą metodę, która przecież ma konstruktywny charakter, ile użytek, który zrobił z niej Cantor. Za rzeczywisty cel metody przekątniowej uznał tam dowód niemożliwości ułożenia w ciąg liczb niewymiernych. Jego zdaniem błąd polega tu na tym, że pojęcie ciągu nieskończonego zostało wyabstrahowane z ciągu liczb naturalnych i innych podobnych ciągów. Niestety nie wystarcza to, by nadać jasny sens pytaniu, czy zbiór liczb rzeczywistych można uporządkować w ciąg. Co najwyżej moglibyśmy stwierdzić, że mamy przykłady porządkowania liczb naturalnych, wymiernih czy algebraicznych i że są to pewne analogiczne wytwory nazywane ciągami. Nie mamy jednakże pomostu pozwalającego przejść od tych przypadków do przypadku „wszystkich liczb rzeczywistych”. Nie dysponujemy także żadną ogólną metodą pozwalającą na ustalenie, czy dany zbiór można ustawić w ciąg. Dla Wittgensteina nie ma wystarczająco jasnej różnicy w sposobie używania terminów „pierwiastek”, „liczba algebraiczna” etc. a pojęciem „liczby rzeczywistej”. Jego zdaniem nie ma sensu mówienie o ciągu wszystkich liczb rzeczywistych, skoro metoda Cantora także liczbę przekątniową tego ciągu określa jako „liczbę rzeczywistą” (por. UPM II, s. 16). Krótko mówiąc, Wittgenstein widział w dowodzie przekątniowym raczej metodę czy też regułę dla sukcesywnego konstruowania liczb, które są różne od każdej kolejnej liczby w danym systemie (ciągu)³².

Drugie zastosowanie metody przekątniowej, które krytykował Wittgenstein, to jej wykorzystanie jako dowodu, że zbiór liczb rzeczywistych jest nieprzeliczalny. Właściwie cała krytyka teorii mnogości Cantora w II Części UPM zaczyna się od tego problemu. Czytamy tam: „10. Nic nie znaczy powiedzenie: ‘*A zatem* liczby *X* nie są przeliczalne’” (UPM II, s. 10). Wedle Putnama stwierdzenie to jest jednym z koronnych dowodów na to, że w II części UPM mamy wyraźne i oczywiste odrzucenie tezy o nieprzeliczalności zbioru liczb rzeczywistych i całej teorii Cantora. Putnam pisze: „Powiada on nam [Wittgenstein – P.D.], ku mojemu zdziwieniu, że *nic nie znaczy* powiedzenie o pewnej klasie liczb *X*, że jest nieprzeliczalna!”

³² Por. Floyd, Mühlhölzer, 2020, s. 140–141.

(Putnam, 2007, s. 236). Jednakże bliższe przyjrzenie się sprawie, co zawdzięczamy szczegółowym badaniom Felixa Mühlhölzera³³, nie potwierdza oceny Putnama. Chodzi bowiem o to, że Wittgenstein w § 10 akcentuje słowo „*A zatem*” (w oryginale – „*Also*”), pisząc je kursywą. To właśnie owo „*A zatem*” nic nie znaczy, a nie to, że „liczby X są nieprzeliczalne”. Słowo „*A zatem*” wyraża bowiem konsekwencje metody przekątniowej, o której mowa we wcześniejszych paragrafach 8 i 9. Wittgenstein natomiast „nieprzeliczalność” poniekąd utożsamiał z tą metodą w tym znaczeniu, że kto ją zrozumiał, ten zrozumiał, że dla dowolnej listy rozwinięć dziesiętnych liczb rzeczywistych istnieje liczba, *sc.* liczba przekątniowa, która nie znajduje się na tej liście: „Pojęcie liczby X nazywam nieprzeliczalnym, jeżeli ustalono, iż którekolwiek z podpadających pod nie liczb ułożymy w ciąg, liczba przekątniowa tego ciągu również ma pod nie podpadać” (UPM II, s. 10). Pomijając niezręczność językową mówienia o nieprzeliczalności pojęcia liczby X zamiast jej ekstensji, „*A zatem*” rzeczywiście nic nie znaczy, ponieważ kiedy je wypowiadamy, to powtarzamy po prostu to, co jest już wiadome z samej przytoczonej wyżej definicji nieprzeliczalności. Albo: „*nic nie znaczy*”, gdyż wedle metody przekątniowej liczby rzeczywiste są nieprzeliczalne. Wittgenstein ma więc rację, przyjmując oczywiście jego definicję nieprzeliczalności. Akceptował on nieprzeliczalność, w tym oczywiście nieprzeliczalność liczb rzeczywistych, ale rozumiał inaczej niż powszechnie rozumie się to pojęcie w matematyce, to znaczy po prostu jako coś, co „nie daje się przeliczyć”. Jak zauważa Mühlhölzer³⁴, matematycy nie zaakceptowaliby rozumowania Wittgensteina, ponieważ teoria mnogości dostarcza ogólniejszej definicji nieprzeliczalności, którą można w pełni zrozumieć niezależnie od metody przekątniowej Cantora. Metoda ta właśnie *dowodzi* nieprzeliczalności liczb rzeczywistych. Wtedy „*A zatem*” oznaczałoby, że Cantor pokazuje nam, że pojęcie liczby X podlega pod ogólniejsze pojęcie nieprzeliczalności, z którym ta metoda nie jest już koniecznie połączona. Przeciw temu dokładnie Wittgenstein oponował w § 10. Jego zdaniem pojęcie nieprzeliczalności powinno być rozumiane w bezpośrednim nawiązaniu do metody przekątniowej, ale wówczas metoda ta nie dostarcza nam nowego faktu matematycznego, ale jest jedynie wynalezieniem nowego pojęcia.

³³ Por. tamże, rozdz. 7.

³⁴ Por. tamże, s. 155.

Gdyby metoda przekątniowa ograniczała się do pokazania, że pojęcie „liczby rzeczywistej” bardziej różni się od pojęcia liczby naturalnej niż zwykliśmy sądzić, to byłoby to jasne postawienie sprawy. Ale, zdaniem Wittgensteina, dzieje się tu coś przeciwnego: zbiór liczb rzeczywistych porównuje się pod względem wielkości ze zbiorem liczb naturalnych, a ich odmiennosc przedstawia się jako różnicę zakresów. „Wierzę i mam nadzieję, że przyszłe pokolenia będą śmiać się z tego hokus-pokus” (UPM II, s. 22)³⁵. Hokus-pokus nazywa tutaj Wittgenstein dowód oparty na metodzie przekątniowej, że nieskończony zbiór \mathbb{R} ma większą ekstensję niż pozostałe zbiory nieskończone. Jest to kluczowy moment w jego krytyce Cantora. Jeśli metodę przekątniową mielibyśmy uważać za dowód na istnienie mniejszej bądź większej nieskończoności, to byłby to, jak mawiał Wittgenstein, „dowód chełpliwy”, czyli taki, który pokazuje więcej niż pozwalają na to stosowane w nim środki³⁶. „Niebezpieczeństwo zwodniczości poglądu, że ‘Liczby rzeczywiste nie można uporządkować w ciąg’ albo, że ‘Zbiór ... nie jest przeliczalny’, polega na tym, że określenie pojęciowe, konstrukcja pojęciowa, jawi się w jego świetle jako fakt przyrodniczy” (UPM II, s. 19).

Wittgenstein był zdania, że różnica między zbiorami skończonymi i nieskończonymi nie jest różnicą zakresów (*resp.* stopnia), ale różnicą kategoryalną. Nie jest on zatem przeciw nieprzeliczalności jako takiej, ale przeciw jej prozatorskiej interpretacji. Powiedzenie, że metoda Cantora *pokazuje*, że zbiór liczb rzeczywistych jest *większy* niż zbiór liczb naturalnych, jest właśnie owym hokus-pokus. W kontekście jego intencjonalnego rozumienia zbioru nieskończonego żaden w ogóle dowód na większą bądź mniejszą nieskończoność nie jest *ex definitione* możliwy. Istotniejsze jest jednak to, że w teorii Cantora \aleph_0 , podobnie jak \aleph_1 , traktuje się jako liczby i porównuje z innymi liczbami, zarówno skończonymi, jak i nieskończonymi. Wittgenstein skłonny byłby przyjąć, że może istnieć wiele różnych systemów liczb rzeczywistych, ale nie możemy skonstruować jednego wyczerpującego systemu wszystkich takich liczb. Zwracał na to uwagę już w *The Big Typescript*,

³⁵ Warto może wspomnieć, że w pierwszym dwujęzycznym wydaniu *Uwag o podstawach matematyki* zdanie o „hokus-pokus” zostało usunięte, jakby wydawcy wstydziło się słów Wittgensteina, a może uznali je za zbyt obrazoburcze. Zob. L. Wittgenstein (1956), *Bemerkungen über die Grundlagen der Mathematik / Remarks on the Foundations of Mathematics*, red. G.H. von Wright, R. Rhees, G.E.M. Anscombe, Oxford: Basil Blackwell, s. 57/57e.

³⁶ Por. UPM II, s. 21.

kiedy analizował różne rodzaje liczb rzeczywistych takich jak π' , P , F , $\sqrt{2}$. W *Uwagach o podstawach matematyki* pisał zaś w tonie kategorycznym: „*Nie istnieje żaden system liczb niewymiernych – nie istnieje jednak również żaden nadsystem, żaden ‘zbiór liczb niewymiernych’ cechujący się nieskończonością wyższego rzędu [podkr. – P.D.]*” (UPM II, s. 33).

Komentując te słowa, Putnam³⁷ zauważa, że Wittgenstein w arogancki sposób odrzuca coś, co jest centralnym twierdzeniem nie tylko teorii mnogości, ale całej współczesnej matematyki, że mianowicie istnieje zbiór liczb niewymiernych i że jest on nieprzeliczalny. Słowa Wittgensteina należy jednak rozumieć w kontekście jego nie-ekstensjonalistycznego stanowiska, wedle którego liczby rzeczywiste nie istnieją jako homogeniczna całość, ale są wytwarzane w rozmaity sposób, który nie jest z góry określony. W przytoczonym § 33 mowa jest o tym, że geometryczny sposób mówienia sugeruje istnienie jakiegoś „super-systemu” o „wyższej” (sc. aktualnej) nieskończoności, podczas gdy wedle Wittgensteina metoda przekątniowa Cantora pokazuje nam, że nie istnieje system liczb rzeczywistych, to znaczy ich ciąg, zawsze bowiem możemy wskazać na liczbę, która nie należy do tego systemu. W tym sensie nie istnieje „zbiór liczb niewymiernych” (właściwie powinno być: „rzeczywistych”) rozumiany ekstensjonalistycznie jako jednorodna dziedzina liczb, ponieważ dla Wittgensteina zbiory nie są definiowane przez ich elementy, ale na mocy reguł, praw i technik konstruowania tych elementów. Z tego punktu widzenia dziedziny liczb nie są czymś jednorodnym, nie mają wyraźnych granic. Na przykład liczb rzeczywistych nie sposób zdefiniować jako przekrojów liczb wymiernych (Dedekind) *albo* przez rozwinięcia dziesiętne *albo* jako zbiór punktów na prostej³⁸.

4.1. Kryterium zastosowania

Ostatni wątek krytyki Wittgensteina związany jest z problemem stosowalności matematyki. W ostatnim z paragrafów części II UPM pisał on:

62. Nie zajmuję się wykazywaniem błędności rachunków, lecz badaniem tego, co w nich *interesujące*. Badam np. to, w jakiej mierze jest tu jeszcze usprawiedliwione używanie słowa ...; (...) Nie mogę zatem powiedzieć: „Nie wolno się tak wyrażać” albo „To jest absurdalne”,

³⁷ Por. Putnam, 2007, s. 239.

³⁸ Por. Floyd, Mühlhölzer, 2020, s. 33.

albo „to jest nieinteresujące”, lecz: „Zbadaj w ten sposób to wyrażenie pod kątem jego zasadności”. Zasadność jakiegoś sposobu wyrażenia nie sposób uchwycić, nie ogarniając *jego zastosowania*, tego zaś nie dokażemy, przyglądając się tylko jednemu aspektowi jego zastosowania, np. obrazowi, jaki się z nim wiąże”.

(UPM II, s. 620)

To właśnie kryterium zastosowania pokazuje, zdaniem Wittgensteina, że nieprzeliczalność jest „hokus-pokus”. Pyta on: „Do czego można użyć pojęcia nieprzeliczalny?” (UPM II, s. 12). – Do niczego. Autor *Traktatu* stał na stanowisku, że matematyczna „gra językowa” stosuje pojęcia i znaki, których używamy poza matematyką, to znaczy w zdaniach empirycznych, przechodząc od jednych do drugich takich zdań. O ile możemy zastosować do zdań empirycznych pojęcie „parzysty”, np. „musimy podzielić się na dwie równe drużyny”, to nie znajdujemy podobnego zastosowania dla pojęcia „nieprzeliczalny”. Jednym z fundamentalnych przeświadczeń Wittgensteina było to, że sam tylko rachunek staje się matematyką dopiero za sprawą jego pozamatematycznych aplikacji. Co więcej, to właśnie pozamatematyczne użycie znaków nadaje im znaczenie: „Chcę powiedzieć: dla matematyki istotne jest to, że jej znaków używa się również w *cywilu*. To użycie znaków poza obrębem matematyki, a więc ich *znaczenie*, czyni z gry w znaki matematykę”. Istotą filozofii matematyki Wittgensteina był bowiem pogląd, obecny już w *Traktacie*, że zdania matematyczne nie są opisem abstrakcyjnych bytów matematycznych (Frege), ani opisem rzeczywistości empirycznej (Russell, Ramsey), ani nie opisują działań umysłu. Zdania matematyczne to według niego reguły reprezentacji, mają sens *preskryptywny*, czyli są wzorcami przekształcania zdań empirycznych. Na przykład zdanie „ $5 > 4$ ” pozwala na stwierdzenie, że Jaś, który ma 5 lat, jest starszy od Małgosi, która ma 4 lata i wyklucza zdanie „Małgosia jest starsza od Jasia”. Podobnie zdania geometrii są regułami gramatycznymi opisu stosunków przestrzennych przedmiotów empirycznych. W tym kontekście Wittgenstein był przekonany, że teoria mnogości tworzy pojęcia, ale nie ma jasności co do ich pozamatematycznego zastosowania. „Nie istnieje technika *gramatyczna*³⁹ [zmiana tłum. – P.D.], która podsuwałaby nam sposób użycia takiego wyrażenia

³⁹ W polskim tłumaczeniu *Uwag* zamiast „technika gramatyczna” (*grammatische Technik*) jest błędnie „technika matematyczna”, co niestety wypacza sens paragrafu 38. Technika matematyczna użycia wyrażeń teoriomnogościowych oczywiście istnieje.

[jak np. $\aleph_0 < c$ – P.D.]” (UPM II, s. 38). W przeciwieństwie do „ $5 > 4$ ” zdanie, powiedzmy, „ $2^{\aleph_0} > \aleph_0$ ” nie jest żadną regułą przekształcania zdań empirycznych, to znaczy nie ma pozamatematycznego zastosowania i w tym sensie nie wiemy, co z tym zdaniem zrobić. Przy czym to pozamatematyczne zastosowanie Wittgenstein rozumiał bardzo szeroko – jako zastosowanie w naukach empirycznych, ale także w życiu codziennym. Rodzi się jednak pytanie, czy teoria mnogości, która nie ma bezpośrednich zastosowań w realnym świecie, jest jeszcze matematyką? Wittgenstein stawia to pytanie wprost: „Jeśli zamierzone zastosowanie matematyki jest czymś istotnym, to jak ma się rzecz z tymi działami matematyki, których zastosowanie – a przynajmniej to, co za takowe uważają matematycy – jest zupełnie fantastyczne? W rezultacie – jak w przypadku teorii mnogości – uprawiamy jakąś gałąź matematyki, o której zastosowaniu mamy zupełnie opaczne pojęcie. Czyż *mimo to* nie uprawiamy wówczas matematyki? (UPM V, s. 5). Na to pytanie można by odpowiedzieć, że teoria Cantora jest przecież grą prowadzoną za pomocą symboli wedle pewnych reguł, nawet jeśli nie mamy jasności co do ich zastosowania. Odpowiedź Wittgensteina brzmi niedwuznacznie: „Ale jak można dysponować pojęciem, a nie mieć jasności co do jego zastosowania” (UPM V, s. 7). „Na razie nie mają zastosowania, ale być może w przyszłości je znajdziemy, również dla zdania $2^{\aleph_0} > \aleph_0$: Na razie jest ono fragmentem zawieszonej w powietrzu architektury matematycznej i wygląda, powiedzmy, jak architrav, nic go jedna nie podtrzymuje i on sam niczego nie podtrzymuje” (UPM II, s. 35). Jednakże, jak podkreśla Rodych⁴⁰, Wittgensteinowskie kryterium zastosowania wymaga syntaktycznej spójności. Jeśli teoria mnogości miałaby być fragmentem matematycznej architektury, to musiałaby być syntaktycznie spójna. Z uwagi jednak na niemożliwość dowodu absolutnej spójności rachunku matematycznego „zawsze będzie potrzebny jakiś dobry diabeł” (UPM VII, s. 16), aby teoria mnogości mogła być z sukcesem zaaplikowana w realnym świecie⁴¹.

⁴⁰ Por. Rodych, 1997, s. 214–216.

⁴¹ Oczywiście problem stosowalności matematyki oraz podział na matematykę czystą i stosowaną jest u późnego Wittgensteina skomplikowany i wymagałby osobnego opracowania. Według Maddy (1993) dla Wittgensteina tylko matematyka, która ma zastosowania w poza-matematycznej rzeczywistości, zasługuje na miano matematyki, podczas gdy zdania czystej matematyki, pozbawione poza-matematycznych zastosowań, pozbawione są także sensu. W takim ujęciu czysta matematyka byłby tylko grą znakami o podejrzanym obiektywności. Inny pogląd reprezentuje Dawson (2014, s. 4143–4148),

Brak *cywilnych* aplikacji rekompensujemy sobie za to tajemniczością, pewnym intelektualnym powabem, urokiem albo czarem⁴² twierzeń takich jak to, że istnieją liczby większe od nieskończoności, i zamiast od razu tłumaczyć je jako skutek błędnego rozumienia, otacza się je powagą i szacunkiem⁴³. To właśnie paradoksy nieskończoności powołały do życia teorię mnogości, bez nich nigdy by ona nie powstała. Nie może być jednak interesujące coś, co powstało z opacznego rozumienia, to znaczy z opacznego rozumienia nieskończoności jako czegoś aktualnego. Wydawałoby się, że sieć języka, w którą zaplątał się Cantor, jest interesująca, ale to tylko pozór. Coś, co powstało z błędnego rozumienia, nie może być interesujące, to znaczy stosowalne. Tajemniczość i czar są nieistotne, w rachunkach liczą się tylko praktyczne konsekwencje, np. ich aplikacje w fizyce. „Pewne

według którego relacja między matematyką stosowaną i czystą jest u Wittgensteina bardziej zniuansowana. Traktował on bowiem matematykę jako „pewną rodzinę” (UPM VII, s. 33), w której główną postacią jest matematyka mająca poza-matematyczne zastosowania. Czysta matematyka, której brak takich bezpośrednich aplikacji w świecie realnym, może mieć jednak związki z innymi systemami matematycznymi, które mają takie aplikacje i w tym znaczeniu czysta matematyka może być czymś więcej niż tylko grą znakami. Interpretację Dawsona podzielają Pérez-Escobar i Sarikaya (por. Pérez-Escobar, Sarikaya, 2022, s. 1–22), którzy podkreślają kulturowy i socjologiczny sens aplikacji.

⁴² W *Wykładach i rozmowach o estetyce, psychologii i wierze religijnej* Wittgenstein sugerował, że wiele wyjaśnień psychoanalizy przyjmujemy, ponieważ mają dla nas pewien czar czy też urok: „Wyobrażenie świata podziemnego, tajemniczej piwnicy. Coś ukrytego, niesamowitego”. Interpretacje Freuda w szczególnie sposób nas pociągają, ich atrakcyjność wydaje się nieodparta i to nie dlatego, że ich podstawą jest rzetelna wiedza empiryczna, ale ważna jest pewna *postawa*, którą wyrażają. Podobny urok i atrakcyjność ma, jego zdaniem, teoria Cantora: „Np. całkowicie obaliłem dowód Ursella. Ale kiedy to zrobiłem, on odrzekł, że ten dowód ma dla niego pewien urok. Na to mogę tylko odpowiedzieć: »Dla mnie nie ma żadnego uroku. Czuję do niego odrazę«. Por. wyrażenie: »Liczba kardynalna wszystkich liczb kardynalnych«. 38. Por. Cantor pisał, jakie to wspaniałe, że matematyk może w swojej wyobraźni [w umyśle – T] przekraczać wszelkie granice. 39. Zrobiłbym wszystko, by pokazać, że to właśnie ten urok tak nas zwodzi. Jako matematyka albo fizyka wygląda to jako coś bezspornego i dlatego nabiera jeszcze większego powabu. Jeśli wytłumaczymy kontekst tego wyrażenia zobaczymy, że rzecz można by wyrazić w całkiem inny sposób. Mogę ją przedstawić tak, że dla wielu ludzi straci swój urok, a już na pewno straci go dla mnie” (Wittgenstein, 1967, s. 42–43).

⁴³ Por. UPM II, s. 16. Echo tych rozważań odnajdujemy w 412 paragrafie *Dociekań filozoficznych*, w których Wittgenstein pisze o „zawrocie głowy, który pojawia się zwykle, gdy dokonujemy sztuczek logicznych. (O taki sam zawrót głowy przyprowadzają nas pewne twierdzenia teorii mnogości)” (DF § 412).

rozważania mogą nas doprowadzić do powiedzenia, że w jednym centymetrze sześciennym mieści się 10^{10} dusz. Dlaczego więc tego nie mówimy? Ponieważ nie jest to do niczego przydatne. Ponieważ wprowadzie przywołuje na myśl pewien obraz, z obrazem tym jednak nie potrafimy dalej nic zrobić” (UPM II, s. 36). Obraz nieskończoności, który powstaje za sprawą pojęć stworzonych przez Cantora, pozostawia nas jakby w zawieszeniu, to znaczy nie wiemy, jaki jest jego związek z rachunkiem. Na pewno nie jest to bowiem taki związek jak między obrazem ||||| a liczbą 7 (por. UPM II, s. 35). Najlepiej byłoby pozbyć się tego obrazu, w samym rachunku nie ma, według Wittgensteina, niczego nieskończonego. Co więcej, jeśli bliżej przyjrzymy się nieskończonemu ekstensjom, to one po prostu znikają jako nonsens i zostaje sam rachunek.

Uwagi końcowe

Pora odpowiedzieć na nasze pytania postawione na początku o powody tak ostrej krytyki teorii mnogości Cantora, nieporównywalnej z krytycznymi komentarzami Wittgensteina na temat Russella, Fregego, Hilberta czy Brouwera. Wskazałbym na trzy główne motywy:

(i) Można krytykę Wittgensteina tłumaczyć częściowo jego finityzmem, niezgodą na logycyzm w wydaniu Russella i Fregego czy konstruktywistycznym w istocie podejściem do matematyki. Także przecież wśród samych matematyków opór wobec teorii Cantora nie był wcale czymś rzadkim. Wystarczy przypomnieć Leopolda Kroneckera, który traktował Cantora jako „naukowego szarlatana”, renegata i demoralizatora młodzieży. Z kolei Henri Poincaré nazwał teorię mnogości „interesującym przyczynkiem patologicznym” i przewidywał, że „następne pokolenia będą traktowały *Mengenlehre* jako chorobę, przez którą się przeszło”⁴⁴. Tak więc na tle tych negatywnych ocen krytyka Wittgensteina w zasadzie nie powinna specjalnie zaskakiwać. Dziwić może i powinien natomiast jej czas, kiedy teoria Cantora była już dobrze ugruntowaną dziedziną matematyki i odgrywała istotną rolę w wielu jej obszarach. Jednakże dla Wittgensteina takie wewnątrzmatematyczne aplikacje nie były wystarczającym argumentem, by uznać teorię mnogości za coś ważnego, ponieważ nie o takie wewnątrzmatematyczne zastosowanie

⁴⁴ Por. Dauben, 1990, s. 1, 134–137.

mu chodziło, ale o zastosowanie poza matematyką w świecie rzeczywistym. Autor *Dociekań* poważnie potraktował empiryczną aplikację zdań matematycznych. W tej perspektywie twierdzenia teorii mnogości wydawały mu się „światlistymi konstrukcjami pojęciowymi” (UPM II, s. 16), które są w naszym życiu codziennym bezużyteczne. Matematykę traktował jak część ludzkiej aktywności, część tego, co nazywał naszym sposobem życia (*Lebensform*).

(ii) Gramatyczne podejście do zdań matematyki, koncentracja na *prozie*, która jest przecież bliższa codziennym praktykom niż *rachunki*, ujawniała ogromne trudności semantyczne dyskursu o nieskończoności. Teoriomno-gościowa gramatyka Cantora nie była dla Wittgensteina przejrzysta, gdyż wykraczała poza dobrze nam znane codzienne „gry językowe”, w których słowa „nieskończony” używamy najczęściej w sensie negatywnym jako coś, co nie ma końca. Wyprawa w otchłanie nieskończoności aktualnej kończyła się dla niego prozą obracającą się na „jałowym biegu” albo wejściem na gładki lód, po którym nie można chodzić, ponieważ nie ma tarcia. Inną sprawą, trudną i pozostawioną bez jasnej odpowiedzi, jest pytanie, czym miałyby być, wedle Wittgensteina, ów „szorstki grunt” w matematyce, po którym chcemy chodzić.

(iii) To wykraczanie teorii mnogości poza granice „gier językowych” miało też światopoglądowe oblicze. Kto wie, czy nie było ono nieświadomym źródłem niechęci autora *Traktatu* do dzieła Cantora? Henrik Von Wright w eseju *Wittgenstein i jego czasy* pisał o dwóch niezdrowych nawykach myślowych, które wedle Wittgensteina dewastująco wpływały na kulturę jego epoki: teorii mnogości i behawioryzmie. „Gdyby dane mu [Wittgensteinowi – P.D.] było oglądać teorię mnogości w roli podstawy nauczania matematyki dzieci w wielu czy też w większości krajów, bez wątpienia byłby zdęgowany, a może uznałby to za świadectwo końca tego, co zwykliśmy nazywać matematyką” (von Wright, 2000, s. 147). Epokę, w której przyszło mu żyć, Wittgenstein charakteryzował za Spenglerem bardzo negatywnie, jako cywilizację – czas zmierzchu i upadku kultury. Uważał, że duch, w którym pisze, jest inny niż duch wielkiego nurtu cywilizacji europejskiej i amerykańskiej, który „wyraża się w postępie, w budowaniu coraz większych i bardziej skomplikowanych struktur” (PR, s. 8). Jego celem było natomiast dążenie do jasności i przejrzystości. Przyczyn upadku kultury i przekształcenia jej w cywilizację dopatrywał się w dążeniu do wykraczania poza „gry językowe” i budowaniu wyrafinowanych konstrukcji myślowych

w rodzaju teorii mnogości Cantora, które jednak nie są związane z żadną formą życia. Tworzenie takich śmiałych wizji jest przekraczaniem granic tego, co sensowne, granic codziennego użycia języka w danej „grze językowej” opartej na zbiorze podzielnych wspólnie reguł i sposobów życia. Pojęcia teorii mnogości są jakby na zewnątrz, poza domem, poza tym, co zwykle robimy. Tworzą dyskurs, który wiedzie nas poza horyzont codzienności, potoczności, poza to, co jest nam bliskie i znane. Wraz z nim musimy opuścić obszar społecznych praktyk, pozostawić dom i wspólnotę, a to oznacza zerwanie więzi, utratę orientacji i samotność. Filozofowie nieskończoności zabierają nas w podróż w nieznanne, namawiają, byśmy opuścili szorstki grunt tradycji albo byśmy nie słuchali przykazania o efektywnej konstruowalności twierdzeń egzystencjalnych w matematyce. Ale ku czemu mamy zmierzać, gdzie jest to miejsce? Sprzeciw Wittgensteina wobec teorii mnogości brał się nie tyle z obawy przed nieznanym, lecz raczej ze zrozumienia konsekwencji porzucenia naszych codziennych praktyk – braku orientacji i wyobcowania.

Krytyka Wittgensteina wyrażona została w języku, który może budzić kontrowersje, a nawet pewne zaskoczenie. Używał sformułowań wieloznacznych, metaforycznych, skrótowych, których sensu trzeba się często domyślać (w końcu nie były przeznaczone do druku). Bardzo trudno jest oddzielić w jego dyskursie to, co zalicza do prozy błędnej, a co do prozy oświecającej. Trudno na przykład nazwać błędną prozą coś, co okazało się jednym z największych osiągnięć Cantora, czyli jego twierdzenie o nieprzeliczalności zbioru liczb rzeczywistych. Jak podkreśla Bernays⁴⁵ w twierdzeniu, że nieprzeliczalność liczb rzeczywistych jest udowodniona metodą przekątniową, nie ma niczego zwodniczego. Wittgenstein mówił o prozie, za pomocą której matematycy objaśniają swe formalizacje, co robili także Cantor i Dedekind, i postulował pozbycie się jej jako szkodliwego metafizycznego naddatku, ale sam przecież tę prozę uprawiał, w określony sposób interpretując dokonania wspomnianych matematyków. Krytykował ich nie jako matematyk (co sam wielokrotnie podkreślał), jego rozważania nie są *rachunkiem*, ale jedną *prozę* zastępują drugą. Opowieść Platónską opowieścią finitystyczną i konstruktywistyczną. Powracający temat jego krytycznej fugi, problem istnienia nieskończonych ekstensji matematycznych, jest w istocie problemem filozoficznym, a nie matematycznym. Czy będziemy Platonikami, opowiadając się za nieskończonością aktualną, czy

⁴⁵ Por. Bernays, 1959, s. 22.

konstruktywistami, stojąc na stanowisku nieskończoności potencjalnej, jest dla matematyki obojętne. Kiedy mówimy na przykład, że dany ciąg jest rozbieżny do plus lub minus nieskończoności, albo o granicy ciągu a_n przy n dążącym do nieskończoności, albo kiedy definiujemy ciąg jako funkcję, której dziedziną jest nieskończony podzbiór zbioru liczb naturalnych, to jest całkiem obojętne, czy nieskończoność rozumiemy aktualnie czy potencjalnie. Albo też inaczej: dany ciąg jest rozbieżny do nieskończoności, jakkolwiek byśmy jej nie rozumieli. Jeśli matematykę potraktujemy jako rachunek, co postulował przecież sam Wittgenstein, to dla rachunku jest całkowicie bez znaczenia, czy na przykład liczby rzeczywiste uznamy za nieskończone ekstensje czy za reguły. Również definicja zbioru nieskończonego Dedekinda, tak bardzo krytykowana w *The Big Typescript*, nie musi się troszczyć o to, czy będziemy mówić o możliwości czy rzeczywistości jedno-jednoznacznego przyporządkowania podklasy całej klasy.

Czy wobec tego Wittgenstein szedł złą drogą, jak twierdził Putnam? Na pewno szedł konsekwentnie swoją własną drogą prowadzącą bardziej przez teren filozofii, którą rozumiał jako badanie słów co do zasadności ich użycia, a więc jako przedsięwzięcie z istoty pojęciowej natury. Na matematykę patrzył w nieco inny sposób niż matematycy, to znaczy jako na przykład ludzkiej aktywności, tworzenia reguł i wzorców postępowania, które kierują naszym życiem, a nie jak na dziedzinę prawd, które się odkrywa. Jak pisał: „Matematyk jest wynalazcą, nie odkrywcą” (UPM II, s. 168). Kąśliwa uwaga Bernaysa: „Wittgenstein argumentuje [w kwestii nieskończoności – P.D.] jakby matematyka istniała prawie wyłącznie w celach prowadzenia domu” (Bernays, 1959, s. 22) nie byłaby dla niego krytyką, ale w pewnym sensie pochwałą.

Wykaz stosowanych skrótów pism L. Wittgensteina

BT = *Big Typescript: TS 213* (2005). German-English Scholars' Edition. Red. C.G. Luckhard, M.A. E. Aue. Oxford: Blackwell.

D = *Dzienniki 1914–1916* (1999). Tłum. M. Poręba. Warszawa: Wydawnictwo Spacja.

LCA = *Lectures and Conversations on Aesthetics, Psychology, and Religious Belief* (1967 [2007]). Red. C. Berrett. Oxford: Basil Blackwell.

LFM = *Wittgenstein's Lectures on the Foundations of Mathematics Cambridge 1939* (1976). Red. C. Diamond. Hassocks, Sussex: Harvester Press.

- MS = rękopisy Wittgensteina według klasyfikacji Georga H. von Wrighta. Por. G.H. von Wright (1982). *Wittgenstein*. Oxford: Basil Blackwell, s. 32–63.
- PG = *Philosophical Grammar* (1974). Red. R. Rhees. Tłum. A. Kenny. Oxford: Basil Blackwell.
- PR = *Philosophical Remarks* (1975). Red. R. Rhees. Oxford: Basil Blackwell.
- TLP = *Tractatus logico-philosophicus* (1997). Tłum. B. Wolniewicz, Warszawa: Wydawnictwo Naukowe PWN.
- UPM = *Uwagi o podstawach matematyki* (2000). Tłum. M. Poręba. Warszawa: Wydawnictwo KR.
- WLC = *Wittgenstein: Lectures, Cambridge 1930–1933. From the Notes of G.E. Moore* (2016). Red. D. Stern, B. Rogers, G. Citron. Cambridge: Cambridge University Press.
- WWK = *Wittgenstein und der Wiener Kreis. Werkausgabe*. T. 3. (1984). Frankfurt am Main: Suhrkamp.

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ARTYKUŁY

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Ingarden's Aesthetic Argument against Husserl's Transcendental Idealism Turn

Keywords: Roman Ingarden, Edmund Husserl, idealism–realism, phenomenology, aesthetics

Słowa kluczowe: Roman Ingarden, Edmund Husserl, idealizm–realizm, fenomenologia, estetyka

Abstract

Husserl's allegiance to realism came under attack following his *Ideas*. Ingarden was a fierce critic of his teacher's turn to transcendental idealism, and provided compelling arguments both for his idealist reading of Husserl and for his rejection of idealism. One of the main arguments Ingarden devised against Husserl's turn was based on his aesthetics. Against Husserl, Ingarden established literary works and fictional objects as purely intentional objects that are (1) doubly structured, *vis-à-vis* their formal ontology, and (2) endowed with spots of indeterminacy. These facts, Ingarden argues, necessitate the transcendence of the purely intentional object. In this paper, I explore his argument, while establishing the ontological foundation on which it rests.

I

Following his *Ideas*, Edmund Husserl kicked off a hot debate revolving around whether he had taken a turn to *transcendental idealism*. Roman Ingarden, one of Husserl's most prolific students, interpreted the Husserl of *Ideas* onwards as indeed advocating a metaphysically idealist view of the world and its objects, as opposed to his *Logical Investigations*' realist stance.¹ Ingarden, of course, did not change his position and remained a vehement advocate of metaphysical realism. Not only so, Ingarden, troubled with his teacher's turn, devoted almost all his philosophical works to proving Husserl wrong. Therefore, Ingarden's rich phenomenological corpus rests on the idealism–realism debate. Ingarden's aesthetic investigations are also inspired (or provoked?) by Husserl's turn. As we find in the Preface to the first German edition of his *The Literary Work of Art*, Ingarden describes his project thusly:

¹ For the purposes of this article, I will assume Ingarden's interpretation of Husserl is accurate; of course, while presenting Ingarden's arguments in favor of the latter. Many commentators have confirmed Ingarden's reading of Husserl as entailing a turn to transcendental idealism (cf. Haefliger, 1990; Küng, 1993, 1975, 1973, 1972; Makota, 1995). Others have dismissed Ingarden's interpretation of Husserl's idealist turn (cf. Holmes, 1975; Sokolowski, 1977; Wallner, 1987; Hall, 1982; Ameriks, 1977). While I believe Ingarden's idealist reading of Husserl is correct, I do not share his conviction that Husserl started his idealist project in *Ideas*. As is noted by Byrne, Husserl's inclination toward transcendental idealism can already be felt in his *Investigations*. Whereas his 1901 philosophy remains neutral as regards realism/idealism, Husserl's *Second Investigation* unapologetically disavows metaphysical realism, ruling out across the board the existence of mind-independent objects. It also sets forth his method of suspending all reality's metaphysical definitions [cf. Husserl, 1984, p. 129]. Further indications that clearly align with Husserl's transcendental idealism are found in the *Fifth Investigation*, in which a bold borderline is drawn between metaphysical and phenomenological deliberations [cf. p. 401], and the *Sixth Investigation* where he reproaches Kant's metaphysically stained philosophy [cf. pp. 729–732]. Moreover, Husserl's 1906–07 lecture course, "Introduction to Logic and the Theory of Knowledge" [Husserl, 1985], outlines his three "paths" to the *epoché* and the transcendental reduction. It is, therefore, evident that Husserl's transcendental project started way before *Ideas*. The question is, how did Ingarden not pick up on that? (2020, p. 515, footnote 3). This is not, however, meant to undermine Ingarden's interpretation of Husserl, for it is indeed *Ideas* that marks the full maturity of his transcendental idealism.

Although the main subject of my investigation is the literary work, or the literary work of art, the ultimate motives for my work on this subject are of a general philosophical nature, and they far transcend this particular subject. They are closely connected to the problem of idealism–realism, with which I have been concerning myself for many years. (1973, p. IXXII)

Put differently, Ingarden devises his aesthetic findings to counter Husserl's newly held position. To elaborate, Husserl, following *Ideas*, concluded that the world, together with all its objects, is mind-dependent. This, contrary to his realist convictions in the *Logical Investigations*, marks Husserl's adoption of idealism. Writing in his *Motives*, Ingarden explains the idealism–realism division:

The controversy between realists and idealists concerning the existence of the real world is not about the question whether the real world, the material world in particular, exists in general (even Berkeley would protest energetically if somebody told him that he affirmed the non-existence of the material world), but about the mode of the world's existence and what its existential relation is to acts of consciousness in which objects belonging to this world are cognized. (1975, p. 31)

In light of the established realism–idealism debate, Ingarden strived to determine the external world's *mode of being*. In order to bring about a comprehensive ontology of the world and its objects, Ingarden sought to investigate the formal–ontological constituents of various objects. His findings, discussed mainly in *Controversy* (2013/2016), led him to conclude that there are *mind-dependent* objects as well as *mind-independent* objects, contrary to what Husserl's transcendental idealism upheld, which is a depiction of the world and the entirety of its objects being mind-dependent. The objective of Ingarden's ontological deliberations, therefore, can be seen as arguing against the purported *pure intentionality* of the external world. In this paper, I shall formulate Ingarden's aesthetic argument against Husserl's depiction of the world as being purely intentional. Before doing that, I shall first explore Ingarden's idealist reading of Husserl and critically examine his arguments.

II

Ingarden first expressed his skepticism about Husserl's turn toward transcendental idealism in a letter he addressed to his teacher in 1918.² With that said, Ingarden's arguments against Husserl gained full maturity in his *Motives*. This is where Ingarden unleashed the full extent of his arguments against Husserl's turn, and it is this book that will serve as my point of departure. Ingarden structured his book into two main parts: one part presenting Husserl's position and another critically analyzing it. The latter is analyzed in conjunction with four themes: (1) the concept of philosophy as rigorous science, (2) the limits of the phenomenological reduction, (3) the analysis of outer perception and the theory of constitution, and (4) the formal-ontological sources of Husserl's idealist solution. My analysis of Ingarden's reading of Husserl will follow these four points.³

As is well-known, Husserl intended his phenomenology as a *science*, a form of inquiry that can lead to indubitable cognition. Following his *Logical Investigations*, Husserl turned his attention to epistemological issues. Among these, the issue of "outer sense perception" was of special importance to Husserl's project. He believed that outer perception could guide him to the kind of knowledge that would ensure his phenomenology is on a par with science. Soon, however, Husserl concluded that such cognition cannot be achieved by resorting to outer perception. Consequently, indubitable cognition must be pursued some other way:

In these investigations it very soon became evident that outer perception could not yield indubitable cognition. The question then arose if such cognition could be found in inner or, more exactly, immanent perception. This thought must have occurred to Husserl, he was, after all, Brentano's pupil and he introduced the concept of "inner consciousness": by means of it a cognition was to be gained which could not be doubted. Thereby it was suggested that immanent perception was to be analyzed and its cognitive value to be discovered. (Ingarden, 1975, p. 11)

² For details, see Ingarden (1976), pp. 419–438.

³ Mitscherling's (1997) analysis of Ingarden's interpretation of Husserl will be crucial to the general structure of this Section.

Following his recognition of outer perception as a dead-end, *vis-à-vis* his pursuit of indubitable knowledge, Husserl turned instead to inner perception or "immanent perception." To be more explicit, Husserl held that "by means of the initial formal *epoché* [...], we are enabled to attend to the perception itself, regarding the act of outer perception as transcendent to this 'immanent' perception itself" (Mitscherling, 1997, p. 52). Ingarden interprets Husserl's turn to immanent perception as opting for

the directly opposite point of view, treating the sense of the object constituted in the cognitive process exclusively as the creation of the acts coming into consideration. Consequently, he treats the analyzed objects from the beginning exclusively as *intentional correlates of these acts* and these objects have *only* in these acts the source and basis of their existence and such and no other formation of their contents. (1975, p. 37)

This way of going about the issue does not necessarily commit us to an idealist position, as Ingarden makes clear. It would not be an idealism to assert that phenomena are dependent on intentional acts of perception, as long as we can return to the objects "appearing through the analyzed phenomena." But Husserl forbids this return, which Ingarden describes as a major shortcoming in his reduction method (Ingarden, 1975, pp. 37–38). In her (1987) work, Wallner argues the opposite, maintaining that Husserl emphasized time and again the necessity of returning to the objects perceived. For her, Husserl's reduction does not isolate us from the concrete manifestations of the world. "On the contrary, we are 'led back' to the original evidence of the world as experienced and experienceable" (p. 19). Mitscherling (1997) argues against Wallner's line of reasoning on two counts. First, the concrete manifestations of the world, Husserl posits, are to be found in acts of consciousness, and not in in the transcendent physical world, to which, as she claims, the phenomenologist is "led back." Second, Wallner seems to have neglected the significant impact "as" has on both her own formulation and on Husserl's. The world *as* experienced and *as* experienceable is not the same as the real, transcendental world. Put differently, her formulation depicts the world as a "phenomenon of consciousness," the immanent world, not the world of "realities" (p. 54).⁴

⁴ Ingarden confirms this formulation in a lecture delivered in Oslo in 1967 (cf. 1992, p. 280).

In (2), Ingarden (1975) further delves into Husserl's phenomenological reduction. He concedes that Husserl's method is not only useful but necessary, as far as the critique of knowledge is concerned. Nonetheless, Ingarden does not think the phenomenological reduction is indispensable. When it comes to ontological investigations, of various kinds, the phenomenological reduction is not necessary (pp. 40–41). Epistemology as the critique of cognition faces the danger of a *petitio principii*. There are two main aspects of this *petitio principii* threatening the “objectivity” of cognition. The first aspect pertains to epistemology begging the question of the applicability of reasoning's concepts and understanding to an objective, external world, i.e., of the relation between the object of cognition and the act of cognition. The second aspect concerns epistemology, begging the question of the legitimacy of the fundamental reasoning principles used to investigate this very reasoning itself, i.e., of the “objectivity” of the cognition that studies cognition. Husserl was certain he had averted both aspects of the *petitio principii* with his phenomenological reduction. The first aspect of this danger, Husserl maintained, can be solved with the use of his formal *epoché*, or the “bracketing” of existential presuppositions about the external world. With the use of the reduction method, existential presuppositions about the external world are put on hold, allowing only what is phenomenologically given (i.e., immanent to consciousness). The second danger, Husserl held, can be overcome with the use of the “epistemological reduction.” Building on the suspension of existential prejudices (formal *epoché*), the epistemological reduction suspends logical, ontological, and psychological presuppositions. The main advantage of this method is “enabling us to investigate the various levels of cognition through a series of ‘genetic’ analyses, extending to the lowest, most basic level of the fundamental assumptions of reasoning.” As regards the illegitimacy of the relation between the act and object of cognition, the epistemological reduction helps us avert this problem, for the act of cognition becomes itself the object of cognition. Formulated as such, any uncertainties about the act/object relation at this phenomenological level must be directed at an abstract distinction between two aspects of the act of cognition *per se* (Mitscherling, 1997, pp. 55–56).⁵

⁵ Husserl (1982) suggested this formulation, but he left the problem of *petitio principii* in need of further elaboration. It was Ingarden (1921) who offered such an elaboration.

Tymieniecka (1976) points to another danger threatening the validity of Husserl's phenomenological reduction. This danger is related to the second aspect of the *petitio principii*. As Ingarden indicates in his criticism of *Cartesian Meditations*, the foundation of Husserl's phenomenology is infected with an incurable disease:

On Husserl's theory we have to assume the specific nature of the transcendental consciousness in order to conduct the phenomenological reductions leading to the attainment of the level of self-evident cognition, while it is precisely first through the proper practice of the phenomenological reductions that the transcendental consciousness can be revealed in its nature.

This, Ingarden believes, leads to a vicious circle, which Husserl cannot escape. The reason for the latter is that Husserl's phenomenological method rests on the notion of the intentional character of all conscious acts embedded in the phenomenological method. It is the notion of the intentional that results in a conception of a self-sufficient consciousness (p. 249).

Ingarden takes issue with Husserl's notion of the intentional. According to Ingarden (1975, p. 39), Husserl's pursuit of indubitable cognition based on the phenomenological reduction leads to the confinement of all inquiry to the immanent. The question now is, is not what appeared at first sight to be merely a methodological operation now a *de facto* operation, with which the manner and course of phenomenological research is predetermined to be that of pure consciousness?

In (3), Ingarden further deconstructs the above question in the context of Husserl's analysis of perception. In his *Ideas*, Husserl devises the inadequacy of outer sense perception to consolidate his turn to transcendental idealism. In outer perception, material things always seem to present themselves *aspectually*, i.e., from one aspect or another, and we can never be sure if the perceived object really exists, or if it is the same thing that is perceived. In immanent perception, the objects perceived are always (indubitably) fully determined in all their aspects. This is expressed in Husserl's labelling of sect. 46 of *Ideas I*: "*Zweifellosigkeit der immanenten, Zweifelhaltigkeit der transzendenten Wahrnehmung.*"⁶ Even if accurate, this distinction between immanent and transcendental perception does not

⁶ "Indubitability of Immanent, Dubitability of Transcendent Perception."

entail any essential difference in the mode of existence of the objects given in these perceptions. It certainly does not entail what we find in Husserl's labeling of sect. 44: "*Bloss phänomenales Sein des Transzendenten, absolutes Sein des Immanenten.*"⁷ It also does not entail what we find in sect. 55: "*Alle Realität seiend durch Sinngebung.*"⁸ It is incontestable that synthetic intentions are entities of sense, and that these entities are designated by their respective perceptions, but it would be wide off the mark to state that these objects are identical with the "things" appearing in these sense unities; and hence are (things) merely "phenomenal" (1975, pp. 47–48).

Ingarden rightly detects a metaphysical turn to transcendental idealism in Husserl's "constitution." The view of constitution laid out in *Ideas*, as has been stated above, clearly shows Husserl's equation of "thing" with a built "noema-consciousness." Following the phenomenological reduction, one can speak of "reality" as a mere correlate of consciousness. In this view, reality is regarded as a *meaning*, not as a totality of physical objects. With the use of the reduction, we are able to further analyze the world as meaning. What we then unearth is a multi-layered meaning-structure constituted through several acts of synthesis (Mitscherling, 1997, p. 59). Ingarden argues that a heterogeneity-based view of reality and consciousness cannot be given up for a view along Husserlian constitution, for that would commit us to "idealism":

If Idealism be in this meaning tenable, then the theory of constitution would be identical with metaphysics, and a part of it identical with the metaphysics of the external world resp. with the science of nature. I cannot bring myself to agree with this [kind of] idealism. The essential heterogeneity between consciousness and reality (resp. more generally: 'being') I cannot give up. (1976, p. 424)

Although Ingarden backed his reading of Husserl with textual evidence from Husserl's work, there are some philosophers who maintain that Ingarden simply could not fathom Husserl's subtle language. Further, there are some philosophers who understand Ingarden's criticism of Husserl as attributing to him a Berkeleyan idealism. This is, of course,

⁷ "The merely phenomenal being of the transcendent, the absolute being of the immanent."

⁸ "All reality exists through the dispensing of meaning."

a misrepresentation of Ingarden's criticism, for he made clear Husserl's idealism is not the same as Berkeley's:

By this emphasis on the difference of material things in relation to the experiences of perceptions in which they are given, Husserl's idealistic solution is different from other "idealisms" e.g. that of Berkeley. This transcendence is also a certain *formal-ontological moment* of the situation occurring between the real objects and conscious experiences in which they are given, a moment emanating from, for instance, the formal-ontological assertion about the condition for the unity of the whole of objects, and from the assured difference between the essence of lived experience and material things. (1975, pp. 32–33)

Haefliger (1990) defends Ingarden (particularly against Sokolowski and Wallner), and compellingly makes the case for the accurate representation of Husserl's reduction and constitution provided by Ingarden:

The systematic of a constitutive reflection outlined by Ingarden in the 'Idealism-letter' (1918) is taken up again in the later 'Oslo' lectures (1967). As a matter of fact these read here and there like a clarification of the earlier programmatic statements. They thus offer proof that Ingarden had integrated into his philosophy the operation of the transcendental reduction, the general Husserlian distinction between noesis and noema, and in particular the program of a constitutive 'legitimacy reflection'. In particular, however, they show that Ingarden had in no way wrongly understood the special Husserlian concept of constitution as it is presupposed in the framework of a transcendental analysis: Ingarden always made it quite clear that we're dealing here, as we should, with the constitution of noematic 'senses' and not with the 'constitution' (that is to say, the intentional 'creation') of 'things.' (p. 112)

Moving on to (4), where Ingarden explores the formal–ontological foundations of Husserl's idealism. Ingarden works explicitly with two major elements from Husserl's formal ontology. The first is that a *unity* of parts can only be obtained if they share one *essence*. The second holds that consciousness and physical objects have different essences, and for that a unity of the two cannot be obtained (1975, p. 66). We can summarize Ingarden's position as follows. As the aforementioned elements indicate, as far as existential and material ontology is concerned, Husserl can be considered a *pluralist* (and hence distinguished from Berkeley's monistic idealism). The real world,

ideal objects, and consciousness all exist. They, however, differ, *vis-à-vis* their essences, with regards to their mode of being and their material. Armed with the two formal–ontological elements just laid out, Husserl was driven to assert the unattainability of a *whole* combining the three *parts*. Therefore, we wind up with a view of consciousness cut off from the world. As a result, phenomenological investigations should be exclusively restricted to what is given immanently. The phenomenologist is thereby advised to treat all acts of consciousness as having their immanent contents as their objects, outside of which there is nothing. So depicted, Husserl’s idealism is of a unique form (Mitscherling, 1997, p. 63). In Ingardenian terminology, Husserl’s position can be construed as denying the world “autonomy”:⁹

Husserl’s answer is clear and univocal: The material things given in perception and thought in the cognitive acts super-structured over perception are not an autonomous (separate in relation to conscious experiences) sphere of autonomous being in itself; they are only something that exists in its essence “for” the conscious subject performing the perceptive acts. They are only intentional units of sense and beyond that “*ein Nichts*” (nothing). (1975, p. 32)

Ingarden further articulates his stance on Husserl’s exclusion of the real world from the sphere of autonomy:

Reality exists only insofar as it is something ‘in itself’. That for what it can be intended [*vermeint*] is actually irrelevant to it. It is that what it is ‘in itself’ and as such. It is a being completed at all times, and universally determined [*bestimmt*]. There is no indeterminedness in the world, except as indeterminedness of a potency which itself would be *totally* determined. (1976, p. 426)

With this eloquent passage, I conclude my presentation of Ingarden’s idealist reading of Husserl. In the next section, I shall establish Ingarden’s aesthetic argument against Husserl’s turn, which, as the above passage showcases, foregrounds Ingarden’s vehement denial of the objective world as lacking autonomy.

⁹ I will explore Ingarden’s distinction between “autonomy” and “heteronomy” in Section III.

III

If Husserl is right, then all the objects that can be found in the world are purely intentional. In the spirit of scientific research, all it would take to debunk Husserl's hypothesis is to find at least one object that is metaphysically and epistemologically mind-independent, i.e., an object that exists as something over and beyond a pure *intentionale*. Ingarden overdid himself and established multiple objects that are *doubly structured*, with regards to their formal ontology.¹⁰ In other words, Ingarden's investigations helped him reach multiple objects that are both purely intentional and derive their ontic foundation from existent, mind-independent objects. These objects are primarily *art works*. We can talk of the literary work of art, the musical work, the architectural work, the picture, etc.; all these objects have been ontologically analyzed by Ingarden, as to determine their status and relation to both consciousness and the external world.¹¹

One of the main doubly structured objects that Ingarden explored in his works are fictional objects. Fictional objects best exemplify Ingarden's heteronomy/autonomy distinction, which he devised to explain ficta's ontological status. To elaborate, a fictum is a *heteronomous* entity, meaning that it is "an entity which draws its being and its collective stock of attributes from the enactment [*Vollzug*] of an intentional conscious experience, which in a specific integrated fashion is endowed with a content, and it would not exist at all without this enactment" (Ingarden, 2013, p. 113). Put in simpler terms, heteronomy entails that the object to which it accrues exists only as the product of an intentional act, say imaginative act. On its own, a heteronomous entity does not amount to anything. The intentional acts leading to the generation of fictional objects are themselves *autonomous*, for they exist without being dependent on some other entity. That is to say, "an entity (in the sense of any something at all [*irgend Etwas überhaupt*]) exists autonomously (is existentially autonomous) if it has its existential foundation within itself" (Ingarden, 2013, p. 109). It is, therefore, safe to say that fictional entities' being is ontologically grounded by the intentional acts underlying their descriptions. We can understand the purely intentional acts generating fictional

¹⁰ For a study of Ingarden's intentionality and ontology that bears in mind their affinities with Husserl's (and Brentano's), see Chrudzimski (1999).

¹¹ Cf. Ingarden (1989).

entities as imaginative acts. Husserl would agree with Ingarden's imaginative account, at least the Husserl between 1898–1904/05.¹² According to Husserl (1994), an intentional act necessitates a *content*, not a real object (p. 76). This mirrors Ingarden's conviction, which states that intentional acts bring about dependent ficta that do not share the same ontological status as real objects. What Husserl (1979) means by an intentional act's content is the ability to depict an object in various ways, with the object being *intended* as such. In this view, an imagined object is determined by its content (p. 333). Ingarden, too, takes intentional acts as being *ascriptive* in this sense. That is, a fictional object, considering it an imagined object, can be said to possess many properties by virtue of its *intendedness*. A fictum's properties are not the same properties real objects possess. Ingarden (2013) describes fictional properties as being merely "intended" or "allotted." Fictional properties are contrasted with "immanent" properties, which only real, autonomous objects possess (pp. 115–116). When we say that *Anna Karenina is a woman*, we, *à la* Ingarden, do not mean that the fictum possesses *Womanhood* in the same way Angela Merkel, for example, does. The proposition rather means that *Anna Karenina* is intended to be a woman in the fiction, and, *à la* Husserl, it is this content that is at issue when we consider such fictional (intentional) propositions, not a real woman that exemplifies the property in question.¹³

Up until here, Ingarden and Husserl are on the same page. They would both agree that fictional objects are mind-dependent, following their generation as (purely) intentional entities. But a problem arises when we consider ficta to be mind-dependent. If fictional objects are dependent on the imaginary acts that bring them into being, wouldn't that make them *constantly* dependent on those acts? And if the latter is answered in the affirmative, we would end up with problematic ficta that, as Wolterstorff (1980, p. 43) describes them, "flit in and out" of existence. Sartre adopted such an account.

¹² Following Jansen (2005), pp. 121–122; (2016), pp. 69–70.

¹³ This is rather an oversimplification of Husserlian imagination. For a thorough examination of imagination in the early and later Husserl, see Płotka (2020), pp. 37–45. It is worth mentioning Twardowski's (1894) account, in which he distinguishes among an intentional act's "content," "object," and "presentation" (p. 3). The act is linked to its object via its content; the object cannot be reduced to the act. Twardowski's theory influenced Husserl's intentionality. Twardowski also influenced Husserl's attack on psychologism. Not to mention, Husserl wrote a review of Twardowski's *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen* (Płotka, 2017, pp. 81–82).

According to him, our imagination is the source of ficta's life. Once we stop thinking about them, they cease to exist (1991, pp. 177–178). One obvious problem with Sartre's view is what Wolterstorff termed as flitting in and out of existence. The view that ficta can ontologically appear and disappear depending on their being imagined does not sit well with many. This and two other problems pertaining to Sartre's imagination are explored by Thomasson. According to her, Sartre's conception runs contrary to how we ordinarily approach fictional entities, and it complicates our fiction experiences, for Sartre believes our imagining of ficta *recreates* the imagined ficta afresh (1999, p. 22). These problems are indeed thorny for proponents of ficta as imaginary objects. Ingarden's imaginary account of ficta, nonetheless, is fundamentally different. In a way, the problem of ficta as imaginary objects can direct us toward the solution of the idealism–realism controversy. What I mean is, invoking Ingarden, we can both (a) solve the constant dependency of ficta on mental acts and (b) refute Husserl's transcendental idealism.

As regards (a), Ingarden resorts to literary works to avert the problem of constant dependence on mental acts. For him, when an author creates a fictum, they create a fictum within a literary work. These works are comprised of sentences and fictional propositions. These sentences are in turn endowed with what he terms a “borrowed intentionality”:

Both isolated words and entire sentences possess a borrowed intentionality, one that is conferred on them by acts of consciousness. It allows the purely intentional objects to free themselves, so to speak, from immediate contact with the acts of consciousness in the process of execution and thus to acquire a relative independence from the latter. Being purely intentional, the objects “created” by the units of meaning remain both ontically heteronomous and ontically dependent, but this ontic relativity of theirs refers back directly to the intentionality immanent in the units of meaning and only indirectly to the intentionality of the acts of consciousness. (1973, pp. 125–126)

Because they enjoy a borrowed intentionality, sentences and fictional propositions are able to carry the weight of ficta, so to speak. It is, therefore, no longer necessary for mental acts to be directly involved in the subsistence of ficta. Compared to Sartre's, Ingarden's view makes more philosophical sense. Instead of postulating problematic ficta that flit in and out of existence and ascribing to imagination the power of recreating afresh ficta every time they

are imagined, Ingarden puts forth a view of *ficta* that aligns with our intuitions about *ficta* and fiction. Anna Karenina, for instance, does not rely on our imagining her to remain in existence, nor is her subsistence dependent on Tolstoy's generative mental acts. There are numerous books containing Anna Karenina, and these books' sentences borrow Tolstoy's intentionality. In other words, the literary works in which Anna Karenina features are now the link between her subsistence and Tolstoy's conscious acts that first brought the character to life.

The conceptualization of *ficta* as being *derived* is key in Ingarden's phenomenological ontology. The existential foundation of fictional entities, Ingarden posits, is derived. To elaborate, Anna Karenina's "immediate" existential foundation is traceable to the sentences and propositions of the literary works in which she is described. Since sentences and propositions are also purely intentional, they in turn have their immediate existential foundation in the mental acts of an author and a competent reader (cf. 2013, p. 117).¹⁴ This is what marks *ficta* as derived entities, and which, consequently, shields Ingardenian *ficta* from the problematics of merely imaginary objects.

Now about (b). Ingarden's reply to the charge explored above also contains his reply to Husserl's transcendental idealism. This argument is primarily aimed at Husserl's formal-ontological considerations in favor of idealism. As has been explored in the fourth motive that led Husserl to transcendental idealism, his formal-ontological deliberations convinced him that a unity of consciousness and the world cannot be obtained, and that the world should be excluded in favor of consciousness.¹⁵ Ingarden

¹⁴ For more on the role of readers, especially in connection with the concretization of the literary work's aesthetic value, see my essay (2022).

¹⁵ Ingarden believed that, before considering the validity of idealism, we should thoroughly examine the possible relations between the world and consciousness. With this in mind, Ingarden (1976, pp. 435–436) pointed out to Husserl four possible formulations of reality and consciousness, stressing the importance of the separability/inseparability of each's *essence*. Piwowarczyk (2020) argues that, in comparison with Twardowski's, Ingarden's existential conditionings and their opposites as being implied by the essences of the objects they define fare better. The theory of dependence adopted by Twardowski is similar to the one advocated by analytic philosophers. "According to this approach, *x* is dependent on *y* iff it is necessary that *x* exists only if *y* exists." An undesirable consequence of this approach is that it makes objects dependent on all necessary objects. To dodge this problem, it is helpful to treat dependence and existential conditionings in terms of *essence*. Here, Ingarden follows Husserl. Ingarden's take

responds in kind, i.e., his reply is based on his formal–ontological deliberations on intentionality and intentional objects. This is the topic of his § 47 “The Form of the Intentional Object that Corresponds to a Straightforward Act of Meaning” (2016). The objective of Ingarden in this section is two-fold. He attempts to demonstrate that (1) the purely intentional object, *qua* existentially heteronomous entity, does not derive its ontic foundation from consciousness alone, and that (2), *contra* Husserl, there are autonomous entities that exist independently of conscious acts.

Ingarden is concerned here with fictional works, considering them entities that are formally doubly structured. To make manifest this double structure, Ingarden analyzes Rilke's “DAS LIED DES AUSSÄTZIGEN” (the song of the leper). Aware of the complicated nature of this purely intentional object,¹⁶ Ingarden restricts his analysis to only one partial complex, that which forms the “reality” (*Wirklichkeit*) of the work. The reality of the poem is located in the following: (1) the spoken words comprising the poem's text, (2) that about which the words speak (e.g., the leper's relation to his surroundings), and (3) that which is expressed by these words (e.g., the leper's felt hatred). These features jointly constitute the poem's reality. This is what gives us the sense of a self-sustaining (*eigenständigen*) reality, whereas in effect everything about the poem is intentionally projected. What we have here is a mere “figment (*Fiktion*),” put together following various acts of intending (*Intentions-Akte*). The being of the poem is, therefore, the being of a purely intentional (heteronomous) entity, which can be traced back to the (autonomous) intentional acts of Rilke. If one wishes to explore the poem's *actual* properties (formal and material peculiarities), they will find that the creative acts of Rilke are existentially founded in other entities (e.g., in script, in several ideal connections, in conceptual units, etc.). The poem has two “facets.”

on the realism–idealism dispute is embedded in his equation of Husserl's givenness modes with ways of existence. “Combinations of the possible ways of existence of pure consciousness with the possible ways of existence of the world are possible solutions to the realism—idealism controversy.” The idealistic solutions depict the world as existing heteronomously in relation to pure consciousness. The realistic solutions advocate an autonomous world [cf. Ingarden, 2013, pp. 167–226]. Husserl's idealism, therefore, endows consciousness with originality, autonomy, and independence, *vis-à-vis* the world, and the latter with derivativeness, heteronomy, separability, and dependence as regards consciousness [cf. Ingarden, 2013, pp. 180–181] (pp. 539–540).

¹⁶ See Ingarden (1973) for details.

One facet pertains to the “content” in its reality, and the second pertains to the poem as a formally purely intentional object. The latter is characterized by properties that may be *eidetically* reached. One of these properties, for instance, is the property of *content*, which the purely intentional object must possess. With that said, through eidetic analysis, *à la* Husserl, we can only examine what is immanently given in an intentional act; in our case, the structure of the act. This means that we cannot rely on eidetic analysis to further examine the intentional object’s property of content. Thus, eidetic analysis cannot refer the content of the purely intentional object back to the three aforementioned features. In simpler terms, eidetic analysis considers the content (*Gehalt*) of a purely intentional object as being part of the content (*Inhalt*) of a conscious act, which so makes it unable to refer the *Gehalt* to its ontic foundations lying outside consciousness (2016, pp. 206–214).

Husserl’s confounding the content of a purely intentional object with the content of an autonomous object is made explicit when we consider the case of literary works. Adhering to Husserl’s eidetic analysis, the ontic foundations that ground the content of a literary work lie outside the scope of consciousness and would hence be eliminated, but they nonetheless exist. These ontic foundations cannot be restricted by the purported immanence of consciousness. According to Ingarden, autonomous entities are not doubly structured. It is only purely intentional objects (such as literary works and ficta) that enjoy a formally double structure. These objects are, *contra* Husserl, both existentially heteronomous with regards to an author’s conscious acts and derive their existential ontic foundation from autonomous, mind-independent entities.¹⁷ Moreover, Ingarden (2016) explores another component that formally separates purely intentional objects and autonomous objects. Building on his analysis of perception, Ingarden notes the *indeterminate* character of literary works and their objects. Autonomous objects, by contrast, are fully determined in their qualitative endowment. The purely intentional object is, by its essence, full of “spots of indeterminacy” (*Unbestimmtheitsstellen*). This, however, only affects that which is implicitly “co-intended” in the content of an object. The content’s facets established above remain wholly determined. To be more explicit, Rilke’s “the leper,” e.g., can be described in only so many ways. Whatever we say of him would merely constitute one “horizon” of the character, leaving out an infinite

¹⁷ Cf. Ingarden (2016), p. 213.

number of ways in which he can be further described. Spots of indeterminacy, therefore, can be construed as the result of the *finitude* of intentional acts and the *infinitude* of the determinations that can be ascribed to ficta. This, Ingarden argues, leads to a new sense of *transcendence*, in which the autonomous object transcends the cognition that grasps its properties, for the finite acts of cognition can never exhaust them (pp. 214–218). Ingarden proceeds to discuss five concepts of transcendence in §48, but only one of them is particularly related to our current problematic, namely “Transcendence of the Plenitude of Being (*des Seinsfülle*).” This concept of transcendence characterizes the purely intentional object. Here, Ingarden posits that it is the content of the purely intentional object *as a whole* that falls prey to spots of indeterminacy. To elaborate, the purely intentional object’s “plenitude of being,” with its full content and spots of indeterminacy, is transcendent to the individual correlative act of consciousness (p. 221).

IV

In conclusion, Ingarden’s analysis of the literary work’s double structure and spots of indeterminacy illustrates the transcendence of this seemingly mind-dependent object. Ingarden, therefore, not only shows that, *contra* Husserl, there are mind-dependent and mind-independent objects in the world, but also demonstrates that there are objects that exhibit both features, i.e., objects that are both purely intentional and derive their ontic foundation from existentially autonomous entities. The literary work of art and fictional entities, thus, constitute Ingarden’s *aesthetics*-based defense of realism over transcendental idealism.

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Inconsistencies in Himma’s Intellectual Property Theory

Keywords: immaterial goods, intellectual property, state of nature, natural rights, original appropriation, Locke

Słowa kluczowe: dobra niematerialne, własność intelektualna, stan natury, prawo naturalne, pierwotne zawłaszczenie, Locke

Abstract

The intellectual property theory of Kenneth E. Himma aims to vindicate natural rights to abstract objects, believed to form the “intellectual content of creations.” Himma proposes a reformulation of John Locke’s property rights theory in terms of value. He maintains that even if abstract objects preexist their alleged creation, then they are not ready for consumption until access to them is provided by laboring innovators and artists. He declares that making them available is an act of value creation that justifies granting intellectual property rights. In this paper, tacit presuppositions on which Himma’s theory relies are examined and challenged. Against his claims, it is argued that no human labor can improve the availability of abstract objects. It is then demonstrated that “intellectual commons” cannot be “stocked” by human activities and that the alleged value creation cannot happen, because the concept of value is inapplicable to abstract objects. This derails the theory. Finally the meaning of rights envisaged by Himma is investigated. It is shown that they cannot be exercised with respect to causally inert entities.

Introduction

Several scholars have attempted to provide moral justification for intellectual property (IP) rights. A popular strand of these theories refers to the philosophy of John Locke, and portrays IP as a natural right, derived from the pre-institutional, pre-contractual state of nature. One such theory has been proposed by Kenneth E. Himma. It deserves attention for at least two reasons. Firstly, Himma states his views more systematically and explains his assumptions more precisely than many other IP advocates.¹ Secondly, he recognizes shortcomings of the Lockean approach in the context of IP justification, noticing that the nature of abstract entities makes their appropriation more difficult than is usually admitted. In an attempt to overcome this obstacle, he formulates his own, original theory. The present study is an evaluation of his proposal, as exhaustively discussed in Himma's (2012) paper.² It is aimed at answering the question: *Is it possible to justify IP through Himma's theory?* In what follows, it is argued that the answer is negative.

The study is an internal critique. It is carried out by examining the logical structure of Himma's views. In the second section a sketch of his theory is presented and his explicit ontological assumptions on the objects to be regulated are recognized. In the third and fourth section Himma's terminology is examined in more detail. A conflict is identified between his stated explicit assumptions, and a tacit presupposition that abstract objects can be created and controlled. This leads to the realization that *abstract objects to be governed by Himma's proposed rights cannot be created, possessed or used*. In the fifth, sixth and seventh section the consequences of this finding are explored. A refutation is presented for three essential claims of Himma's theory: content unavailability, stocking of the commons and value creation. Possible objections are dealt with in the eighth section. In addition, the proposed meaning of postulated IP rights is examined in the ninth section. It is shown that they are ineffectual. The investigation is supplemented by analogies involving material and immaterial objects.

¹ Such as Nozick (1974), pp. 174–182, Easterbrook (1990), Diamond (2015), or Cwik (2014, 2016).

² Some of his other works are also accounted for (Himma 2005a, 2005b, 2007, 2008).

An outline of the theory

The stated aim of Himma's work is to "provide the beginnings of a viable moral justification for recognizing and providing legal protection of intellectual property." He describes his approach as following the line of arguments inspired by John Locke's theory of property, signaling that what is sought is the justification of objective natural rights. By objectivity he means that the existence and contents of these rights are not social constructs, and their validity does not depend on how many people believe in them. They are meant to be "conferred by morality," rather than by the legal system, and to exist even in the state of nature. At the same time Himma emphasizes that these rights are not necessarily property rights *per se*. He suggests that they might equally well be called "property" or "shmoperty." The objects to be governed by the envisaged rights are referred to as the *intellectual content of creations* or, in short form, as the *content*. Himma states that they are abstract entities "with radically different properties than material or mental objects," and that in particular they lack extension, solidity and spatio-temporal location.³ These entities are "intangible and neither here nor there." They would exist "in a world where there are no minds" to think of them. They cannot be "perceived by any of the five senses" and the way we come to understand them is by reasoning. Himma also maintains that they "can be simultaneously appropriated by everyone" without diminishing the supply available for others. Moreover, he admits that abstract objects are causally inert, so humans cannot interact with them. Elsewhere, he also concedes that they cannot be destroyed.⁴

Himma speaks of IP in general terms, mentioning patent, copyright and trademark systems. Hence the "intellectual content" may represent both technological innovations and works of art. Nevertheless, the emphasis is placed on copyright. He routinely refers to "authors," mentioning sculptures, novels, poems, films, theatrical plays and culinary recipes as examples of objects to be regulated. However, he distances himself from

³ The view that objects governed by IP are abstract is also expressed by Drahos (1996), p. 6; Craig (2002); Resnik (2003); Madison (2012); Von Gunten (2015), p. 12; Błaszczuk (2016) and Chatterjee (2022).

⁴ See: Himma (2005b).

any existing IP regime, declaring instead the interest in “the general issue of whether intellectual property protection is morally justified.”

The argument begins by criticizing the standard Lockean theory of original acquisition. Himma considers two interpretations of Locke’s thought. The first postulates that a property right in previously unowned material objects is acquired by mixing one’s labor with them. According to the second, the improvement of material objects through labor is the source of property rights. Himma notices that both readings depend critically on our ability to causally interact with preexisting resources. Therefore, he finds both of them inapplicable to abstract entities and concludes that they fail to justify IP rights. This realization leads him to propose another variant of the Lockean account. The argument is rephrased in terms of value. Human labor is still central to the acquisition but its role changes. Rather than being mixed with an abstract object or transforming it, labor is now supposed to create its value. Himma maintains that even if abstract objects initially “exist somewhere in the logical space,” then they are not ready for consumption until access to them is provided by the labor of innovators and artists. He interprets this act as value creation and insists that those who “bring new value into the world” should be granted a right “to define the terms upon which others may take advantage of this value.” Therefore, it is the act of making the abstract object available that is meant to justify rights of innovators and artists. Himma envisages some limits to appropriation. He approvingly mentions Lockean provisos but then suggests that a better restriction would be to “balance interests” of all competing parties. This is followed by an admission that the weighing of interests is a “messy, imprecise business” and that Himma does not possess any “sort of algorithm for assessing” them. Instead, circumstances that might facilitate weighing are discussed. Certain “gut-level intuitions” are presented on why particular interests should prevail in various situations. Himma concludes that “some intellectual property protection is morally legitimate.”

Content creation

Himma’s distinction between material “creations” and their immaterial “content” roughly reflects the type-token distinction widely discussed in the

formal analysis of intellectual property theories.⁵ The “content” corresponds to abstract types while “creations” correspond to material tokens of those types. Nevertheless, Himma refrains from mentioning types and tokens, and sticks to his preferred terminological convention. A specific feature of his writing is the insistence on creation, exemplified by the statement:

What is important is whether content creators have an interest in the content they create that the law should protect by allowing content creators to exclude others from the content they create unless the content creator consents to its appropriation.⁶

In this passage, and in the whole paper, readers are literally bombarded with a constant reiteration of two words. Not fewer than 125 occurrences of the word “content” in the paper are accompanied by the reference to creation *in the same sentence*.⁷ The meaning of these unqualified statements is obvious. Himma suggests that the abstract “content” is brought to existence by innovators and artists.⁸ This appeals to a widespread sentiment in favor of granting to individuals property rights in what they have produced. However, besides incessant repetition, no evidence for creation is provided. Meanwhile, these assertions are incompatible with his explicit declaration that abstract objects constituting the “content” would exist “in a world where there are no minds” and with his statements about novels initially existing “in logical space.”

More importantly, Himma's creationism is in conflict with his firm declaration that abstract objects cannot take part in causal relations. If a given object is not capable of causal interaction, how could it be created? The former implies that one cannot affect its mode of being. The latter implies the contrary. Similarly, the creation postulate cannot be reconciled with

⁵ See: Moore (1997, 1998); Dodd (2008); Fallis (2007); Treerise (2008); Wilson (2009, 2010); Wreen (2010); Biron (2010, 2016); Radder (2013); Faraci (2014); Koepsell (2015, p. 52); Uszkai (2014, 2017); Koepsell and Inglott (2017); Hauser (2017); Young (2020), p. 21.

⁶ See: Himma (2012), pp. 1106–1107.

⁷ For the record, 12 mentions of “content” are accompanied by the verb “discover” and 11 by the verb “produce” in the same sentence.

⁸ Similar claims are made by LeFevre (1971), p. 68; Rand (1986), p. 141; Gordon (1989); Yen (1990); Becker (1993); Reisman (1996), p. 388; Ramello (2005); Attas (2008); Peterson (2008); Spinello and Bottis (2009), p. 9; Breakey (2010); Spitzlinger (2011); Lambrecht (2015) and Timmermann (2017).

Himma's concession that abstract objects are indestructible. This implies again that one cannot affect the existence of the abstract "content." But in order to create, one must be able to affect it. These two statements are again self-contradictory. Causally inert, indestructible entities cannot be created.

Finally, Himma (2005a) admits that abstract objects are unique, which means that there are no identical-but-distinct abstract objects.⁹ This may be confronted with properties of the time-space. According to Einstein's Theory of Relativity, the chronological ordering of two events A and B may vary between different reference frames.¹⁰ Such frames may be found when the two events occur outside event horizons of each other, and in particular when they are distant and simultaneous in one frame. Let events A and B correspond to two individuals respectively coming up with the same "content." It may happen that in some frame A precedes B, while in another frame B precedes A. Temporal orderings of alleged originations contradict each other. Each contender is overtaken by someone else. Hence none of them is the creator. Creation does not take place. This argument extends to any abstract "content," irrespectively of events' time-spatial configurations.¹¹ Hence, contrary to Himma's claims, the "content" is not created.¹² What humans can create, are barely its material embodiments.

Possession and use

Other verbs also require attention. It is sometimes claimed that immaterial objects may be *possessed*.¹³ However, the possession is simply the ability to direct a given object: to manipulate and rearrange it. It does not depend

⁹ There is an important external reason to do so. Uniqueness is dictated by the principle of parsimony.

¹⁰ The reference frame is a four-dimensional time-spatial coordinate system.

¹¹ There is no reason to believe that initial existence or nonexistence of an abstract object could depend on later events distributed throughout the whole time-space. See Gamrot (2021) for more details.

¹² This is pointed out by Tucker (1926), p. 286, Luper (1999), and Dodd (2000) for various kinds of types.

¹³ The possession of ideas is mentioned among others by Spooner (1855); Hettinger (1989); DeLong (2002); Moore (2003, 2004, p. 159, 2012, 2015); Sandefur (2007); Kraft and Hovden (2013).

on history and reflects actual physical possibilities, rather than duties or rights defined by a positive or moral law.¹⁴ Now, from Himma's admission that abstract objects are causally inert, it immediately follows that their mode of being cannot be altered. Therefore, they cannot be controlled, and they cannot be possessed. Metaphorical statements about possessing the "content" may be paraphrased in such a way that they refer to material or mental entities. They typically represent being aware of an abstract object, being able to draw others' attention to it, or to produce its material representations. This may involve rearranging matter or mental states. But it does not entail any ability to affect abstract objects anyhow.¹⁵ The "content" cannot be possessed.¹⁶

Another action often mentioned with respect to immaterial beings is *using* them.¹⁷ Himma repeatedly refers to the use of abstract content. The verb "use" reflects putting something into service, employing that thing for a purpose, or attaining an end by means of that thing. All these characterizations involve a purposeful action with respect to an object and the goal that is attained by changing the state of the object. However, being causally inert, as explicitly conceded by Himma, abstract objects cannot be affected by any action and cannot cause anything. Hence the talk about using them is meaningless. They are passive. They may be thought of, spoken of, identified, described, embodied, considered or recognized, but they cannot be directed to fulfill human purposes.¹⁸ Various statements on the "use" of "content" merely reflect mentioning abstract objects, pointing to them, referring to them or using their material representations. These metaphors may be paraphrased in a way that does not imply causality. The "content" is not

¹⁴ It is therefore not synonymous to ownership and should not be conflated with it. See: Demsetz (1967); Bouckaert (1990); Kinsella (2003); and Narveson (2010).

¹⁵ This is recognized by Scanlan (2005).

¹⁶ Błaszczuk (2020) also notes that declarations of possessing intangibles are devoid of any real content.

¹⁷ See e.g., Hughes (1988); Kuflik (1989); Gordon (1993); Weckert (1997); Friedman (2000), p. 138; Damstedt (2003); Spinello (2003); Lemley (2004); Epstein (2009); Breakey (2009); Yung (2009); Murphy (2012); Cernea and Uszkai (2012); Cohen (2014); Varelius (2015); Lester (2016); Slutskiy (2021), p. 235.

¹⁸ This is noted by Reinach (1989), p. 53, cf. Massin (2017). Similarly, Boldrin and Levine (2008), p. 173 recognize that "no usable non-rival knowledge ever came into existence." The finding agrees with Wiśniewski's (2020) view of ideas as preconditions of action, rather than economic goods.

used. The consequences of these findings are explored in the following sections.

Availability for consumption

Apart from suggesting creation, Himma offers another justification for IP rights, whereby abstract objects have to be “made available” by expending labor before they are “consumed”. He explains:

Of course, these propositional objects might have already existed as abstract objects in logical space prior to their creation or discovery, but the important, interesting, nonobvious propositional objects cannot be readily consumed by people until someone, through the expenditure of her labor, makes it available to other people.

and states:

The proof of Fermat’s Last Theorem, for example, did not become available for consumption, despite the intense labors of mathematicians for hundreds of years, until Andrew Wiles produced it in 1994. *A Tale of Two Cities* did not become available for consumption until Charles Dickens produced it. Although it might be true that someone else would have eventually found a proof for Fermat’s Last Theorem, it is not true that someone else would have written *A Tale of Two Cities* had Dickens not done so. The probability of someone else independently composing a perfect copy of what is *A Tale of Two Cities* is so low as to be morally negligible.

Are these claims credible? Could abstract objects really be consumed? Understood narrowly, the verb “consume” represents ingesting a given object. In a wider sense it may also mean using this object, expending it, depleting it, or using it up. But abstract “content” is immaterial, so it cannot be ingested. Being causally inert it cannot be used for any purpose, expended, depleted or used up. Hence it cannot be consumed in a narrow or even wider sense. Himma’s reference to consuming the “content” amounts to a category error. His “availability for consumption” claims are hence pointless. What could perhaps be consumed is a material “creation” but not its abstract “content.” Claims of “content consumption” make as much sense as declarations that a distant galaxy is consumed by using up

or ingesting its photos or descriptions. The galaxy may have ceased to exist billions of years before its distant glow reaches Earth.

The related assertion that “important, interesting, nonobvious” abstract objects are “not available” unless someone labors on them is also unconvincing. The abstract “content” cannot be controlled, possessed, used, or consumed and no amount of human labor can enable these actions. One might then ask, what else could it mean for such an object to become readily available or accessible? Himma seems to suggest that there is some barrier, mechanism, or law of nature that prevents anyone from thinking about it or embodying it unless an inventor or artist expends some significant amount of labor. However, there is no such barrier. Elsewhere, Himma admits that “a poem or a song might be created in a matter of minutes with little thought or effort.”¹⁹ It is also widely recognized that technological or artistic innovations may emerge in a blink of an eye, spontaneously, or accidentally, without any spectacular great-barrier-crossing activities. Very often it is enough to notice the existence of various innovations that appear in nature without any human involvement. These include suction cups, echolocation, hook-and-loop fasteners, anesthetics, anticoagulants, and antibiotics. Himma cannot deny that these are important and interesting. They are also nonobvious until they become widely known. Moreover, the world is replete with objects which have extraordinary aesthetic qualities, and which were not created by humans but may be instantly noticed, appreciated, photographed, and publicized. They constitute glaring counterexamples to his nonavailability claim.

Even Himma's example of Dickens's novel fails to support his rhetoric. In the case of a novel, the abstract type reflects a sequence of signs (words, or perhaps letters and punctuation marks). Due to limited human capabilities there must be some practical limit to its length. It may be very high, but it is *finite*. Hence for a given alphabet of signs and length limitation, we may consider the set U of all the possible sequences—that is all possible novels. Knowing the alphabet size one can easily count them according to well-known combinatorial formulae. Let N be the count. It may be astronomically large, but it is certainly finite as well. Our ability to count these sequences implies that they already exist.²⁰ Moreover, by running a random

¹⁹ See: Himma (2012), p. 1139.

²⁰ Those who claim otherwise would have to explain what is it that is counted.

number generator one may draw any element from U in such a way that all its elements are equally likely to be selected. Contemporary computers may instantiate the corresponding novel in mere fractions of a second. Those who happen to draw the sequence that corresponds to Dickens's novel will have it instantly embodied: displayed, printed or saved to a hard drive. This is not contradicted by the observation that Dickens spent several months writing the manuscript. Undeniably, the first sequence chosen (likely some meaningless noise) is perfectly accessible and available without any spectacular effort. But the situation is symmetric. Each sequence is equally likely to be drawn, regardless of being interesting or not. Therefore, the accessibility status of all the sequences—that is novels—must be the same. Contrary to Himma's suggestions, they are all perfectly available for instant embodiment. This reasoning is not affected by the magnitude of N . It remains equally true when N reaches 10^{10} , 10^{100} , 10^{1000} or any other impressive number. Himma's arbitrary declaration that the probability of choosing the same sequence is "morally negligible" is irrelevant. It does not contradict symmetry.

These findings may run against popular intuitions. After all, Dickens did not use a random number generator to prepare the manuscript of his novel. The chances of instantly becoming a new Dickens, Verne or Tolkien through random draws are miniscule. This apparent paradox is resolved by distinguishing between two categories: likelihood and availability. Himma apparently treats them as synonyms. But their meaning is different. One does not imply the other. There are numerous situations where they diverge. Let us consider some material examples:

Example 1

An automated fragrance warehouse offers N distinct perfume varieties. In order to have a sample delivered, one just needs to type in an identifier of a given substance that is a number between 1 and N . An individual A spends a long time assessing various aromas, in order to find the finest one. For a very large N , the probability that another individual B will draw randomly without any guidance the perfume chosen by A equals $1/N$ and is very small. The chances that B will instantly select another equally attractive substance may also be modest. Finding such a sample may require numerous attempts. But all those observations cannot contradict a simple

truth. Every substance in a warehouse, including the finest samples, is equally, instantly available. It is this fact that makes A's search practical.

Example 2

Two individuals, A and B, possess teleport devices which let them visit any place on the surface of Mars. The individual A spends years making numerous attempts to find a location which offers a particularly impressive panorama. As Mars is a huge planet, the probability that B instantly lands without guidance in the same place is extremely small. Most likely B's first landing will end up in a boring Martian desert covering most of the planet's surface. It may also take several attempts to find any aesthetically interesting place at all, unless B is particularly lucky. Despite these observations, it cannot be denied that a teleport makes all the locations equally, instantly accessible.

The condition of unavailability is logically distinct from, and does not follow from the low likelihood of being independently or spontaneously chosen. What an author actually does, is just an evaluative selection.²¹ A sequence of signs is chosen for embodiment in matter from among a huge multitude of equally available possibilities. The choice involves more or less extensive, direct or indirect comparisons among various sequences. Material embodiments of the selected sequence—such as physical books—may be used for reading pleasure or for other benefits. Depending on the skills, knowledge, expertise and effort of the author, the choice is more or less likely to bring these benefits. Nevertheless, contrary to Himma's allegations, the abstract sequence is not made any more available when its embodiments are distributed. Nothing prevents thinking about it at any moment. All sequences are perfectly accessible for anyone willing to perform an evaluative selection or a random draw. The fact that most of them will never be chosen due to the massive number of possibilities is irrelevant. Analogically, one does not make a number—say 79495498—any more available by uttering it, transmitting it or writing it down. These acts may draw someone's attention to it, but they do not make it more accessible than all other numbers are. Our ability to think of this number does not depend on someone having already written it down. It does not depend on someone having planted 79495498 carrots in a garden either, even if Himma declares that the probability of others

²¹ See: Dodd (2000).

independently doing the same is “morally negligible.” The view of abstract objects as initially “non-available” is groundless.

Stocking of the commons?

Himma mentions “commons” several times when discussing the original Lockean theory in the context of appropriating material objects. In these statements, the meaning of the word seems to reflect the set of preexisting unowned material resources, in general agreement with Locke’s writings. However, he then focuses on “intellectual commons” and states:

The intellectual commons, unlike the land commons, is not a resource already there waiting to be appropriated by anyone who happens to be there; it is stocked by and only by the activity of human beings. Although people can improve the value of land, they cannot make land; in contrast, people can and do make novels, music, proofs, theories, et cetera, and if someone does not make a particular novel, it is not available for human consumption, even if it exists, so to speak, somewhere in logical space.

The reference to “stocking the intellectual commons” suggests supplying abstract objects or providing them. But these objects have no location, so they cannot be transported from elsewhere in order to declare “stocking.” Moreover, Himma’s concession that they preexist their making in the “logical space,” rules out the possibility that they were indeed made. Entities that already exist cannot be “made” or “created,” because these verbs signify transition from non-being into being. An object that exists, cannot undergo this transition again. Finally, as shown in the previous section, the abstract “content” cannot be made any more available or accessible by human labor. All of this means that the “stocking” claim cannot be reconciled with Himma’s assumptions, and in particular with the lack of causal interaction. It is self-contradictory.

The source of this incoherence may be traced back to an equivocation. It is possible to interpret “novels, music, proofs, theories” as abstract objects (say, a novel is an abstract sequence of signs), or as material objects (say, a novel is a particular gathering of paper and ink). Himma conflates these two meanings in a single argument. In order to resolve self-contradictions,

the type-token distinction must be recognized. It is abstract types that exist “in logical space” before anyone thinks of them. But it is only their material tokens that are “made,” “created” or “made available.” The abstract “intellectual commons” cannot be stocked with abstract “content” by human activity. What humans can stock, are material shelves in material libraries, and material warehouses where material embodiments of types are stored.

Evaluating value creation claims

The central component and main innovation of Himma's IP theory is the value creation postulate. This is supposed to be the missing link that connects intellectual labor with causally inert abstract objects. Himma explains:

[...] Locke does not exploit what seems a plausible moral principle that could form the basis for a second, more persuasive version of the argument—the moral principle that when a person labors on an object from the commons and creates new value in it, that person is entitled, as a matter of both fairness and just desert, to the value he creates and hence the object.

So, it is the increase in value of a given object that is supposed to justify rights to it. Instead of mixing labor with an abstract object or improving it, Himma suggests that making this object “available for consumption” increases its value and hence vindicates innovators' or artists' IP rights.

This argument is easily refuted. Abstract objects constituting the “content” are like numbers.²² The claim of making them available for consumption is pointless, because they cannot be consumed. It is also impossible to make them available for thinking, because they always are. This is enough to conclude that Himma's argument fails to justify IP. However, his theory also collapses at a more fundamental level. The concept of value is simply inapplicable to such abstract entities.

Firstly, being causally inert, these objects cannot be used for any purpose. Their mode of being never changes. They cannot be directed to attain any objective. Therefore they have no *use value* and no amount of human

²² Moglen (1999) notes that many abstract objects regulated by IP simply *are* numbers.

labor can change this state of affairs. The alleged value creation cannot follow from making them usable or useful.

Secondly, these objects cannot be controlled or possessed. Therefore one cannot acquire them or give up possession. They cannot be subject to transactions.²³ Hence it is nonsensical to assign exchange ratios or prices to them. This means that they have no *exchange value*. It does not exist and hence cannot be modified to announce any value creation.

Thirdly, there is no way for innovators and artists to create *intrinsic value* in these objects. This is because the intrinsic value is the value that the entity has “in itself,” “for its own sake,” “as such” or “in its own right,” and not because of someone’s actions. The intrinsic value cannot be implanted or embedded in an object. This is precisely what the word “intrinsic” means. Moreover, these objects cannot be created, so the intrinsic value cannot be created along with them. If they are intrinsically valuable at all, then they are so timelessly. If they are not, human actions cannot make them so.²⁴

The above considerations may be summarized as follows. Abstract objects that constitute “content” have no use value or exchange value. If they have any intrinsic value, then one cannot change it. Hence any declaration of creating value in them is nonsensical. Himma’s variant of Lockean theory fails to justify IP rights.

Possible objections

The finding that one cannot “create value” in the abstract “content” may trigger resistance. Certain possible counterarguments are dealt with below.

One intuitive objection would point to situations where individuals are paid for informing others. Indeed, weather forecasters, mining prospectors, detectives, teachers, scientists, and spies are routinely paid for extending the knowledge of other individuals. Music composers, screenplay writers, film directors and advertising specialists are paid for proposing various ideas to be implemented. For numerous occupations, something that might be called a “market for information” apparently exists, and one might be tempted to

²³ This is noted by Gamrot (2022).

²⁴ This is not contradicted by Himma’s discussion on the intrinsic value of time. Time is not the “content.”

conclude that the abstract “content” may be traded, priced or valued. This view is wrong. All these prices are not paid for abstract objects. None of these individuals gives up the possession of an abstract object and no one acquires such a possession. The prices are simply paid for the *service* of making others aware of various facts or types, or drawing others’ attention to them. The provision of such services may change mental states of those served, or the state of their notepads, and laptop screens. However, it does not affect abstract “contents.” Analogically, an astronomer who is paid for studying distant galaxies only sells the service of informing others about them, but does not sell galaxies. The amount of money earned this way is not the price of a galaxy.

Another possible objection might point to IP-related contracts signed every day. They involve selling manuscripts, novels, films, patents, computer programs and shares of IP-reliant companies. Hence it might be tempting to conclude that the prices paid reflect the value of associated abstract “content.” This is also untrue. These contracts depend on regulations of positive law that allow certain individuals to prohibit certain uses of paper, ink, computers, printers or human bodies. In contracts, an innovator or artist is simply paid for not interfering with others’ use of their own material resources. Again, abstract objects are not affected. Analogically, an astronomer who discovers a new galaxy could be legally authorized to prohibit others from pointing their telescopes towards it. This could be enforced, and the violation could be harshly punished. Some clever legislator might even call it “intergalactic property.” But such a regulation would only affect the use of material resources on Earth. Any price the astronomer might negotiate for letting others legally see the galaxy, would not be the price of that galaxy. It would be the price of waiving a regulatory barrier in the use of their own material devices.

The meaning of IP rights

Like many other IP advocates, Himma distances himself from existing legal regulations. He prefers to concentrate on vague general principles, leaving the details to others. Arguing for “some protection” is a clever strategy. It lets him condemn and reject existing absurdities as resulting from incorrect implementation of allegedly just—although never precisely

stated—principles. A vague postulate of “some protection” may always signify something different than the currently discussed perversion. This, however, will not shield his general claims from the critique. Let us now consider the meaning of proposed IP rights.

According to Himma, authors should be granted rights to “control the disposition of the content they create” and the “authority to exclude others from appropriating those contents.” He reiterates these two incantations several times and in several papers.²⁵ But the abstract “content” is causally inert. Its state cannot be changed. It cannot be controlled anyhow or disposed of anyhow. The proposed right to control the disposition of such an entity is hollow. Moreover, humans cannot create such objects. So the rights to “created content” refer to nothing. The set of abstract objects to be governed by these rights is empty.

The postulate of “excluding others from appropriating” does not fare better. Appropriation is an action which amounts to taking a given object into possession or ownership. Possession is simply the factual ability to direct the object and ownership is the right to an undisturbed control over it.²⁶ In other words, it is the right to use, possess and dispose of an object.²⁷ But the abstract “content” cannot be controlled, used or disposed of. Hence there is no meaningful way in which it could be possessed or owned. This rules out any appropriation. The action Himma wants to regulate is impossible. Therefore, the right to exclude, that he speaks of, is pointless. There is no way to exercise it.

Essentially, claims about “controlling the disposition of the content” and “excluding others from appropriating the content” make as much sense as the talk about controlling the disposition of distant galaxies and excluding others from appropriating them. They are groundless declarations devoid of any practical meaning. The actions that may indeed be undertaken with respect to abstract objects include recognizing them, thinking of them, and remembering them. But any prohibition regarding such objects requires an individual to engage in these very actions: to recognize them, think of them and remember them. Prohibiting these actions would be self-contradictory.²⁸

²⁵ See: Himma (2005a, 2007, 2008, 2012).

²⁶ See: Resnik (2003), Kinsella (2009).

²⁷ See: Mossoff (2005).

²⁸ See also: Penner (1997), p. 120.

Blatant inconsistencies are resolved by realizing that Himma equivocates again. He speaks of controlling the abstract "content" and excluding others from it. But what he means is the control over material objects already owned by others.²⁹ Himma simply postulates a partial redistribution of their material property rights. However, types and tokens are separate entities and rights to them are logically distinct. The right to control matter does not follow from rights to abstract objects, just like rights to control a house do not follow from the right to a train, and rights to control boats do not follow from owning a space station. Himma's theory fails on yet another level.

Conclusions

Causally inert, unique entities cannot be created, possessed, used or consumed. Hence the abstract "content" contemplated by Himma cannot be made available or accessible. One cannot boast of having increased its value, to justify appropriation. The "content" cannot be altered or controlled anyhow. Therefore it is also impossible to exercise any rights with respect to it. Consequently, the question posed at the beginning of the present study must be answered negatively. Himma's theory fails to justify IP rights.

This fiasco may be entirely attributed to an equivocation. Himma conflates material tokens with immaterial types tacitly postulating, against his own assumptions, that actions undertaken with respect to material "creations" also affect their immaterial "content." The following analogy exposes the fallacy. Abstract entities resemble unreachable celestial bodies. Whether X is a distant galaxy or a piece of abstract "content," the following statements remain true:

1. Identifying, recognizing or apprehending X is not the *creation* of X
2. Producing a material representation of X is not the *creation* of X
3. Being aware of X is not *possession* of X
4. Possessing a material representation of X is not *possession* of X
5. Using material representations of X is not the *use* of X
6. Consuming material representations of X is not *consumption* of X

²⁹ This is noted among others by Palmer (1990), Kinsella (2008), Dominiak (2014) and Wysocki (2014).

7. Being able to prohibit material representations of X is not *ownership* of X
8. Learning about X and representing it in matter is not *appropriation* of X
9. The value assigned to material representations of X is not the *value* of X
10. The value assigned to the service of informing others about X is not the *value* of X

The disregard for these distinctions resulted in fatal inconsistencies that derailed Himma's IP theory. Those who intend to propose a convincing and logically correct IP justification may wish to take them into account.

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