MAŁGORZATA MAKIEWICZ

MATH & art

VISUAL WAYS TO REPRESENT SYMBOLIC MATHEMATICAL CONCEPTS

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MATH & art

INTRODUCTION

The medium is the message.

Marshall McLuhan

The book, prepared in the convention of visual alphabetisation, is the result of over twelve years of *research in action* on identifying the thought processes that occur when combining photographic art with verbal coding in the field of culture and mathematical education. This research was made possible by the implementation of the International Research and Teaching Project *Mathematics in Focus*[1] – since 2010 at the University of Szczecin, and since 2019 also at the Maria Grzegorzewska Academy of Special Education in Warsaw. Internalising mathematical knowledge by means of interpreting photographs with titles referring to mathematics and externalising one's own mathematical knowledge by means of photographic images and assigning verbal codes of meaning to them according to Douve Draaisma's concept of dual coding[2] encouraged me to develop a concept for teaching mathematics called mathematical photoeducation[3]. My research was preceded by hermeneutic analyses of more than one hundred thousand works (photographs and their descriptions) that were submitted to a photographic competition between 2010 and 2021.

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1 International Competition *Mathematics in Focus*, www.mwo.usz.edu.pl.

2 D. Draaisma, *Machina metafor. Historia pamięci*, translated by R. Pucek, Aletheia, Warsaw 2009, p. 31.

3 M. Makiewicz, *O fotografii w edukacji matematycznej. Jak kształtować kulturę matematyczną uczniów?*, SKNMDM US, Szczecin 2013, pp. 69–116.

The book is a literal and metaphorical inversion of the 2018 monograph *Math & Art. Enactive Representations in Mathematics Education – Research in Action*. The reversal of the graphic layout and used colours is intended to direct the reader's thinking: to change the previously applied scheme that led from the tools of expression (straws, light, potter's wheel) to the mathematical abstract (polyhedron, cycloid, solid of revolution). This time I selected 11 mathematical objects from different levels and ranges of mathematical knowledge, to which I assigned the titles of individual chapters. The final chapter presents chosen metaphors that aptly approximate the meaning of selected mathematical objects. The symbolic representations highlighted in the individual chapters are brought closer with photographic exemplifications. In this way, different, alternative ways to understanding mathematical concepts or regularities are shown.

Through a photograph taken to reveal one's own individual path of understanding, the level of symbolic representations was approximated. *Iconic representation serves to construct and reconstruct cognitive classes even at the pre-operational level.*[4] The presentation of several different photographs assigned to the same mathematical concept is intended to induce the reader to carry out *a reconstruction of the formal composition of the photographic image and thus enable the generation*, as Ralf Bohnsack writes, *of a foreground interpretative framework*.[5] In sociology, as Piotr Sztompka points out, *the decoding and interpreting of existing photographic material by means of sociological categories, models and theories* [...] *leads to the question about the presented implications*.[6] Similarly, in the educational sphere, the application of an interpretive framework to a cognitive photograph supports the production of a mathematical conceptual grid in the mind of the recipient. Photography provides an excellent means of supporting the transfer between the world of ideas and the world of the concrete. This transfer is bi-directional: from the eye to the mind (interiorisation) and from the mind to the eye (exteriorisation). *At the boundary between what is thought and what is seen, not only is there an interpretation of the elements of the perceived environment, but also the experience of changes in one's own cognitive structures. Experiences are the foundation of the student's mathematisation process, they lead to constructing one's own cognitive network, to the internalisation of knowledge at a higher level through cognitive conflicts*.[7] *If it were not for experiences drawn from the spatial-kinetic zone* [...] *conceptualisation would be completely impossible*.[8] The experiences described in the book and their observations captured in photographs are intended to sensitise the reader to the beauty and omnipresence of mathematics, as well as to initiate pedagogical reflection on culture and mathematical education in the age of media and images.

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4 J. Bruner, *Kultura edukacji*, translated by T. Brzostowska-Tereszkiewicz, Universitas, Kraków 2010, p. 216.

5 R. Bohnsack, *Dokumentarna interpretacja obrazu – w stronę rekonstrukcji ikonicznych zasobów wiedzy*, in: S. Krzychała (ed.), *Społeczne przestrzenie doświadczenie – metoda interpretacji dokumentarnej*, Wydawnictwo Naukowe DSWE, Wrocław 2004.

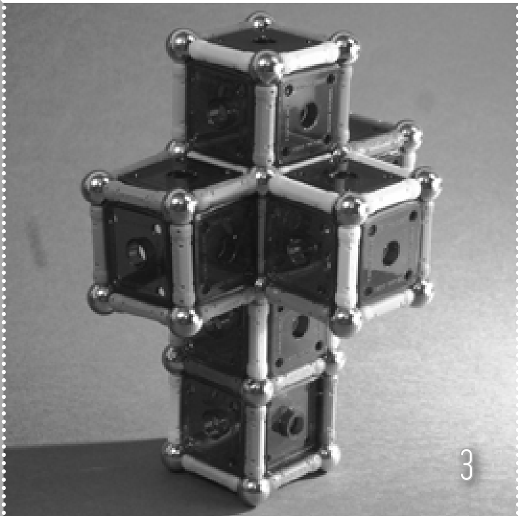
6 P. Sztompka, *Wyobraźnia wizualna i socjologia*, in: M. Bogunia-Borowska, P. Sztompka (ed.), *Fotospołeczeństwo. Antologia tekstów z socjologii wizualnej*, Znak, Kraków 2012, p. 28.

7 M. Makiewicz, *Poznawcza sieć matematycznego myślenia*, SKNMDM, Szczecin 2012, pp. 9–10.

8 A. Gemel, *Od metafory do matematyki. Kognitywna teoria metafory a model nabywania dokładnych reprezentacji numerycznych*. "Studia z Teorii Wychowania", Vol. VIII, 2017, No. 4 (21), p. 93.

1 HYPERCUBE

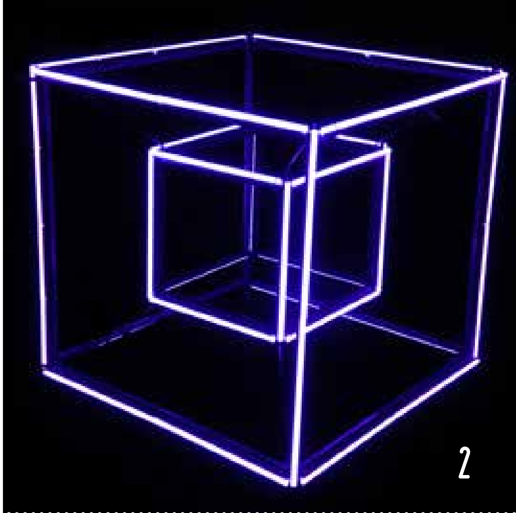
Salvador Dali in *The Crucifixion* showed his idea of a hypercube.[9] He completed an incomplete cube grid in 4D (photo 3, photo 5) with the figure of Christ, thus giving the work a higher-than-third dimension. A hypercube was presented in an interesting way by the photographers, who used the play of light and movement (photo 1, photo 2). In the image (photo 4), we see the cube grid – as a projection of a tesseract. Stefan Banach had a special appreciation for the role of analogy in mathematical reasoning. Let us therefore apply this line of reasoning to an element of four-dimensional space: *The two-dimensional equivalent of a cube is a square. If we move a square perpendicularly to its plane by the length of its edge, then when we connect the corresponding vertices, we obtain a cube. We can obtain a hypercube in the following manner: we move the cube perpendicularly to the space in which it is located by the length of its edge and connect the corresponding vertices*.[10] The longer exposure time applied in photo 1 to the cube put in motion made it possible to mimic the thought process involved in the construction of the cube by shifting the square in parallel and connecting the corresponding vertices. Because, as Jakub Dziewit wrote: *Photographs do not have to be a representation of the world around us, but can also be a projection of the photographer's inner world outwards*.[11]

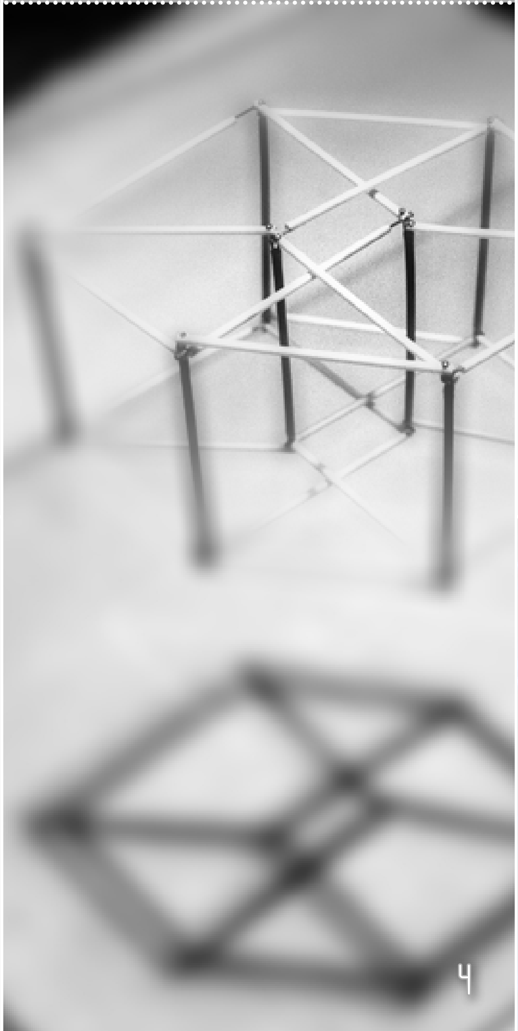


9 The original work is in the Metropolitan Museum of Art in New York, www.metmuseum.org/art/collection/search/488880.

10 K. Ciesielski, Z. Pogoda, *Matematyczna bombonierka*, Demart, Warsaw 2016, p. 85.

11 J. Dziewit, *Aparaty i obrazy. W stronę kulturowej historii fotografii*, Grupakulturalna.pl, Katowice 2014, p. 78.





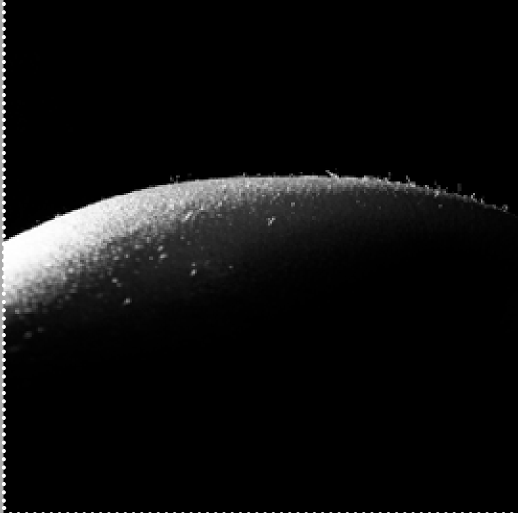
When a non-mathematician hears about the fourth dimension, he gets goosebumps.

Albert Einstein

The idea of four-dimensional space has since its inception been surrounded by an appealing aura of mystery.

Harold Scott Coxeter

2 INFLECTION POINT



The hypercube illustrations shown earlier present the power of human imagination. In them, the artists' fantasy has been developed into imagination leading to highly organised knowledge. The educational significance of such a representation and the difficulty of mentally overcoming the barrier of vision is highlighted e.g. by Terry Barrett: *imagining some scientific discoveries is as difficult as imagining a four-dimensional object in our three-dimensional perception*.[12] Let us move on to a concept directly related to the derivative of a function at a point, denoting a critical event, the moment when the "convexity" of the graph of the function changes. The point on the graph where the convexity of the function changes (from convex to concave or vice versa) is called the inflection point of the function. At the inflection point, the tangents change their position in relation to the curve: from above the curve to a position below or vice versa. Exteriorisations of the mathematical knowledge of the authors of the photographs refer to the natural world: to the body of a woman (photo 1), nature shaping the fruit (photo 2), the seashore (photo 3) or river meanders (photo 4). Photo 2 shows the point of inflection in the work of a luthier, in the shape of the guitar soundboard. The various metaphors of the inflection point show the visual sensitivity and mathematical culture of their authors, allowing them to *open their eyes more widely, to reasonably perceive the regularities, their beauty, elegance, harmony*.[13]

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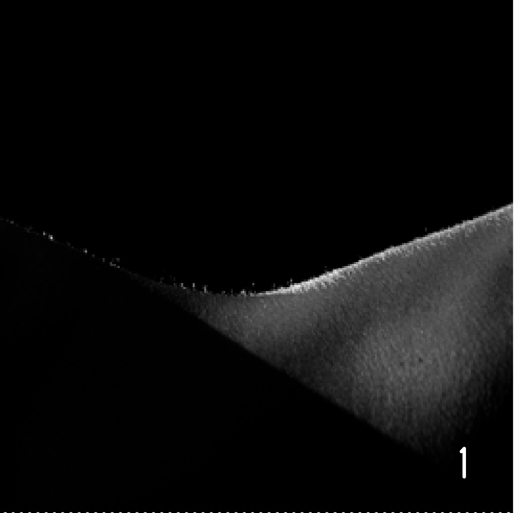
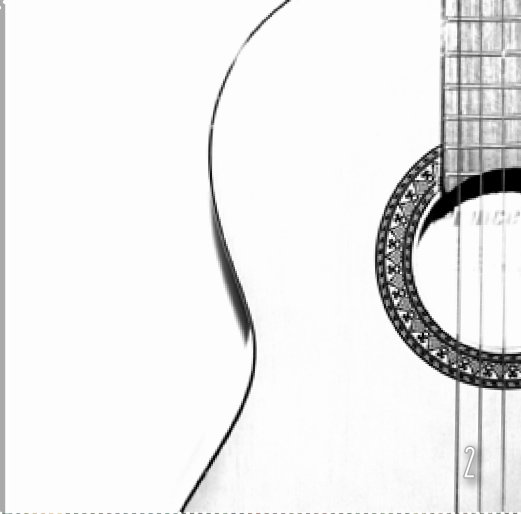
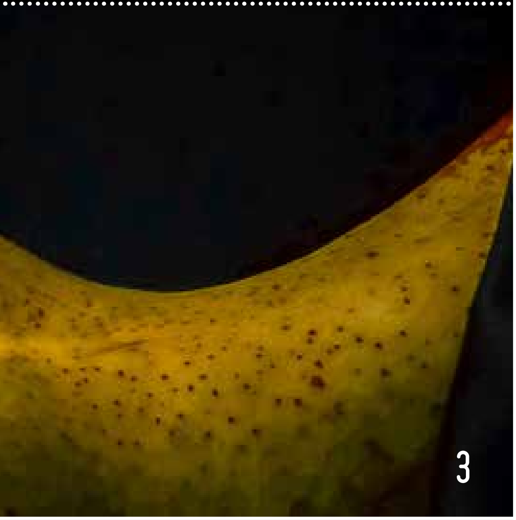
12 T. Barrett, *Krytyka fotografii. Jak rozumieć obrazy?*, translated by J. Jedliński, Universitas, Kraków 2012, p. 48.

13 M. Makiewicz, *Elementy kultury matematycznej w fotografii*, WNUS, Szczecin 2011, p. 7.







*If you change the way you look,
the things you look at change.*

Joe Vitale

3

FIBONACCI

Photographic metaphors of consecutive numbers being terms of the Fibonacci sequence (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …) combine both the ordinal and arithmetical aspects of a natural number. The recurrence relation of the sequence determines the n^{th} term of the sequence as the sum of the two preceding ones. The analysis of the sequence of quotients of this sequence's consecutive values leads to the divine ratio. *The photograph itself denotes what is visible in it: a can, a packet of pasta, spaghetti, mushrooms, peppers, etc. Barthes explains, however, that this image connotes several other messages.*[14] Each of the photographs (photos 1–4) denotes real objects, events. It also leads to cognitive connotations. In the case of the photograph taken in Zawiercie (photo 1), *The Jurassic Fibonacci Sequence*, we can read both a reference to the ammonite – the symbol of the Kraków-Częstochowa Upland, and a golden spiral in the shape of the square. The photograph depicting an unusual arrangement of flowers (photo 2) refers to the cardinal aspect of a natural number. The numbers expressing the powers of individual subsets – flower arrangements – also express the values of consecutive expressions of the Fibonacci sequence from the third to the sixth. The photograph of the digital clock display (photo 3) by means of digits: 1, 1, 2, 3 encodes the time: *eleven twenty-three*. From the description of the photograph we learn of the author's intention to refer to four consecutive expressions of the Fibonacci sequence. The photograph entitled *Waiting for Fibonacci* (photo 4), on the other hand, shows a fragment of a golden spiral shaped by nature.

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14 T. Barret, *Krytyka fotografii…*, p. 56.

Numbers rule the world.

Pythagoras

4

FRACTAL

The authors of the photographs externalise their own perception of self-similar objects called fractals: as a romanesco cauliflower (photo 1), a snowflake (photo 3) or a wild carrot inflorescence (photo 4). In this way, they enable others to follow their own way of thinking. The author's title given to photograph 3, *Winter Loves Us*, reveals the wider context and the author's knowledge of the discoverer of the fractal curve – Helge von Koch. The image speaks much faster than words. Its perception almost immediately triggers a process of interpretation. The area and scope of this interpretation depends not only on the image itself and its title, but also on the personal experience and knowledge of the viewer. It also depends on the context and the extent to which the viewer is guided towards a cognitive interpretation in mathematics. Suggesting the mathematical context of a photograph – e.g. by a teacher – allows to extract from the real image those features that are characteristic, repetitive, that bring the real object closer to the abstract construction. André Bazin emphasises the special role of the objectivity of photography, which *gives the image the power of credibility* [...] *Photography benefits from the fact that the reality of the object is transferred to its reproduction*.[15] In this way, it is possible to shorten the distance between the mathematical abstract and its real representation.

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[15] A. Bazin, *Ontologia obrazu fotograficznego*, in: M. Bogunia-Borowska, P. Sztompka (ed.), *Fotospołeczeństwo…*, pp. 424–425.

Clouds are not spheres, mountains are not cones, coastlines are not circles, tree bark is not flat, and lightning does not move in a straight line.

Benoît Mandelbrot

5 SPIRAL

Photographs are discontinuous moments taken out of the continuum of time.

Terry Barret

Many people remain in awe of the harmony and beauty of spirals. The Archimedean spiral can be seen in works of goldsmiths and blacksmiths (photo 2), as well as in the arrangement of grooves on a vinyl gramophone record. Its essence is the constant distance between successive windings and the direction: from the inside out. The most common in nature, e.g. in shells, ammonites (photo 4), are equiangular spirals called logarithmic spirals. These spirals curl towards the centre, and the distances between their coils are described by a geometric sequence. It is a paradox that mathematics – beautiful, useful, universal – for many is a subject of fear, of anguish. *For the vast majority of students, a particularly difficult and hated school subject.*[16] The exteriorisation of mathematical knowledge by means of photography serves to reverse the unfortunate, but still popular order of teaching. The idea is to precede creating abstract constructs on the basis of symbolic representations by sensory experience, through enactive representations, or by realistic photography referring to iconic representations, which, according to Jerome Bruner, are *intended to perpetuate the specificity of events and objects and initiate prototypes of classes of events*.[17]

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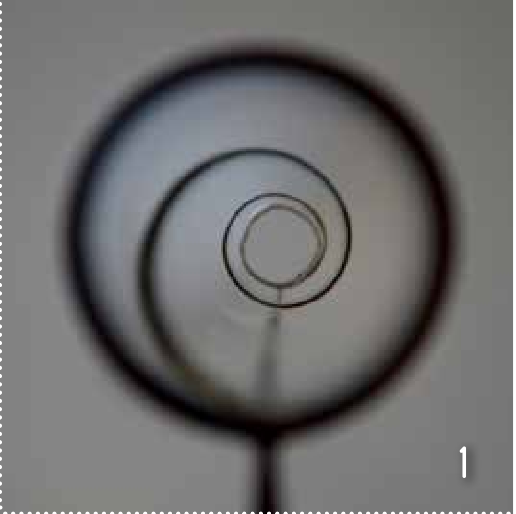
16 D. Gierulanka, *O przyswajaniu sobie pojęć geometrycznych*, PWN, Warsaw 1958, p. 5.

17 J. Bruner, *Kultura edukacji*, translated by T. Brzostowska-Tereszkiewicz, Universitas, Kraków 2010, p. 215.



Eadem mutata resurgo.

Jacques Bernoulli

6 PARALLELISM

Mathematics is the queen of all sciences, truth is its favourite, and simplicity and obviousness its attire... Mathematics, which has done so many favours to society, the sciences and arts, will yet become the leader of the human mind in all cognition.[18] The title of *queen*, which, thanks to Jan Śniadecki, has been ascribed to mathematics, does not give it an elitist character. On the contrary, this unique *queen* serves other sciences. It helps to describe and understand: social processes, structure of matter, regularities of the financial market, economy, medicine, even art. That is why it is so important to support building a system of mathematical knowledge of pupils and students based on the regularities of logic and the structure of conceptual classes. The seemingly obvious concept of parallelism, described as early as 300 B.C.E. by Euclid, is an excellent topic for training conceptual understanding. Selected photographs act as stimuli that evoke a set of *constantly changing readings*.[19] Projecting the surface of the glass roof of the British Museum transforms spherical triangles into *Euclidean* figures. Light transforms the image of spherical parallelism into parallelism in the Euclidean sense. The interior of the balloon (photo 2) refers to parallelism in the system of parallels and meridians on a sphere. The staircase (photo 3) and vineyards (photo 4) illustrate the understanding of the parallelism of curves.

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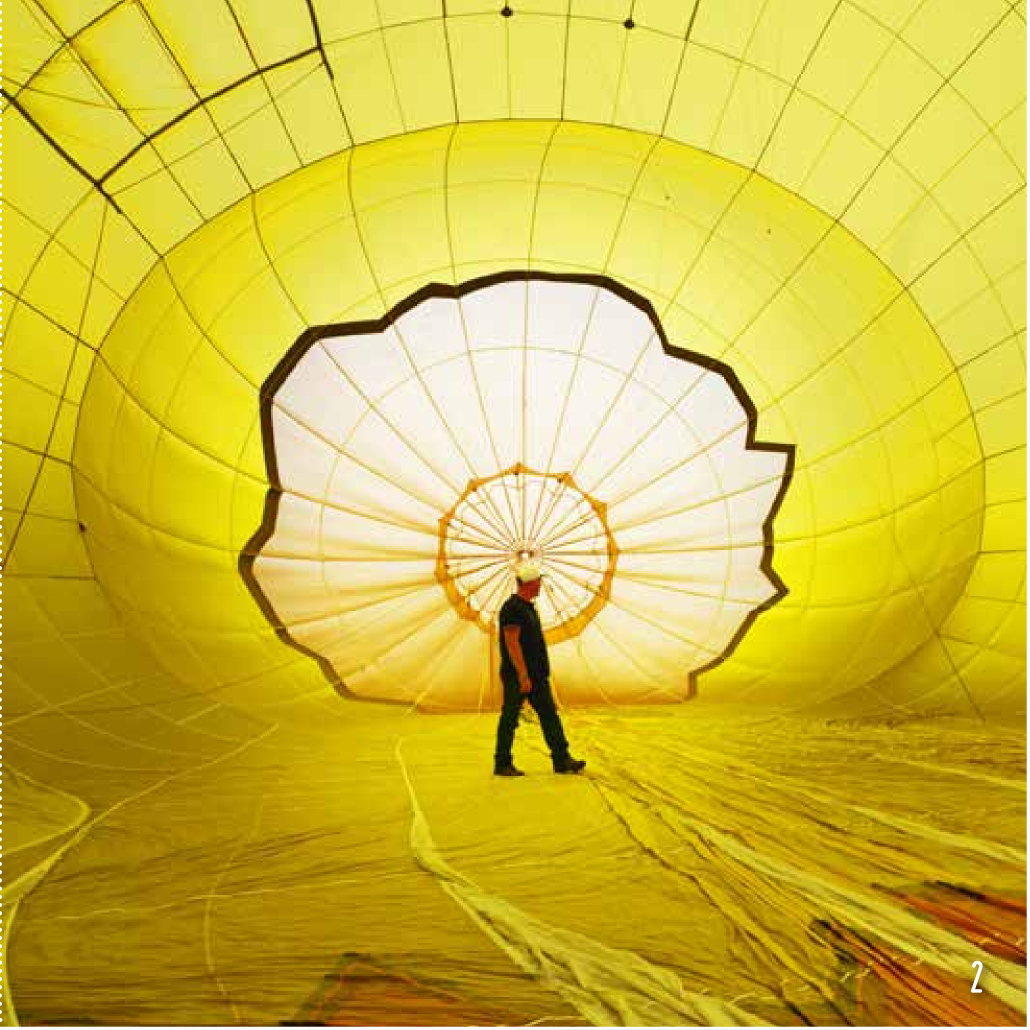
[18] J. Śniadecki, *Pisma rozmaite Jana Śniadeckiego*, Vol. 3, Wilno 1818.

[19] U. Eco, *Dzieło otwarte. Forma i nieokreśloność w poetykach współczesnych*, translated by J. Gałuszka, L. Eustachiewicz, A. Kreisberg and M. Olesiuk, Wydawnictwo WAB, Warsaw 1972.



The subject of mathematics is so serious that it would be useful not to miss the opportunity to liven it up a little.

Blaise Pascal

There is no royal road to geometry.

Euclid

7

ADJACENT ANGLES


1

A proper understanding of the concept of adjacent angles is often problematic. Adjacency is associated with proximity and lack of distance, but the second condition imposed on the angles that we are to call adjacent (their sum is a straight angle), is not intuitive. Various photographs relating to observed events externalise the authors' knowledge and provoke the observers' reflection about the meaning of the concept. Pierre Bourdieu draws attention to a particular kind of photography that is an expression of *ephemeral experience, a momentary infatuation, curiosity or a random event*.[20] Cameras, commonly used thanks to smartphones, allow the observer to react immediately and capture momentary events, light impressions (photo 3), a deserted cliff seashore (photo 5). *Images can be produced quickly by almost anyone. They are highly reliable and extremely transparent*.[21] Each feather in the peacock's tail (photo 1) is an illustration of the common arm of angles complementing each other to a straight angle. The symmetrical reflections of the arms of the skylight (photo 2) show families of adjacent angles in the realisation of the architect's idea, while the common arm of adjacent angles, marked by a fragment of a Greek island protruding over the coastline, is emphasised by the line of the sailing rope (photo 3).

No clock will show you tips for life.

Stanisław Jerzy Lec


4

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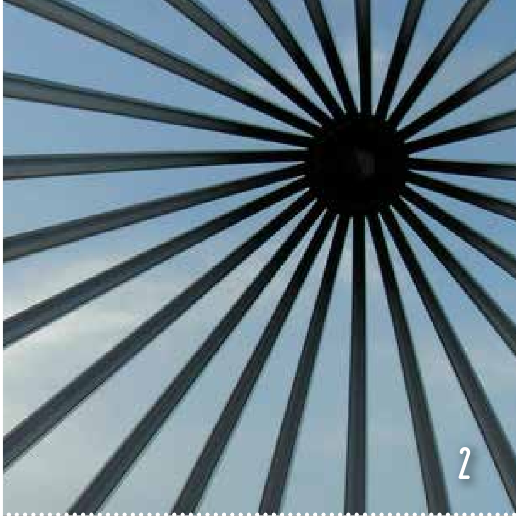
20 P. Bourdieu, *Społeczna definicja fotografii*, in: M. Bogunia-Borowska, P. Sztompka (ed.), *Fotospołeczeństwo*…, p. 242.

21 J. Janowski, *Język wizualny cywilizacji europejskiej*, in: P. Francuz (ed.), *Komunikacja wizualna*, Wydawnictwo Naukowe Scholar, Warsaw 2012, p. 71.

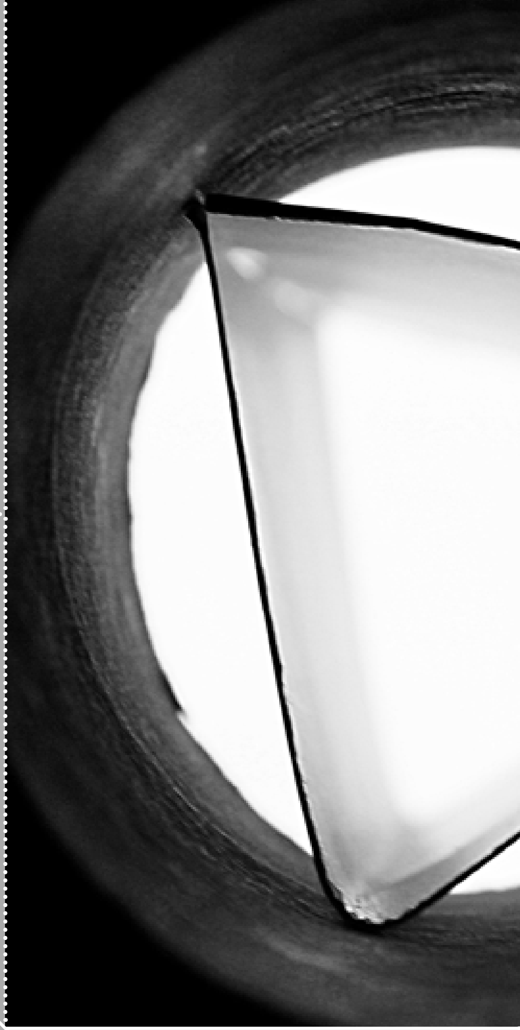
Geometry will draw the soul towards truth, and create the spirit of philosophy, and raise up that which is now unhappily allowed to fall down.

Plato



8

INSCRIBED IN A CIRCLE



In order to introduce a new problem in school mathematics, let us use, similarly as in the case of the hypercube, the mechanism of analogy. Inference on the basis of similarity will make it easier to solve the new problem (inscribing a square in a circle – photo 2) by reducing it to a previously solved one (inscribing a triangle into a circle – photo 1). The similarity refers to the fact that all the vertices of the polygon are on the circle. Gilles Fauconnier and Mark Turner recognise the special role of *identification, integration and imagination in mental cognition. Matching elements from two domains and aligning them with each other, finding a common scheme of their structure that would justify the analogy between them, is now recognised as a great feat of imagination, which computational modelling cannot cope with at this stage* [...], *the ability to perceive analogies in the everyday reality, like the ability to perceive identity, is something that human beings take completely for granted at the level of consciousness*.[22] The photograph reminiscent of the *Vitruvian Man* by Leonardo da Vinci[23] (photo 4) refers to the cognitive sense of the photograph of a red equilateral triangle inscribed in a white circle (photo 3). The double exposure of the figure of the man refers to the aspiration to fulfil the condition of inscribing the figure in a circle.

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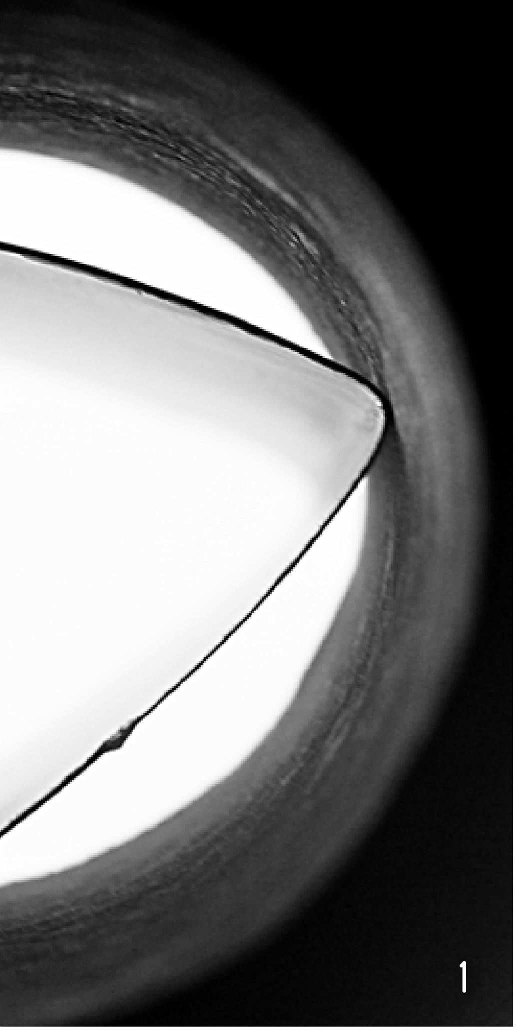
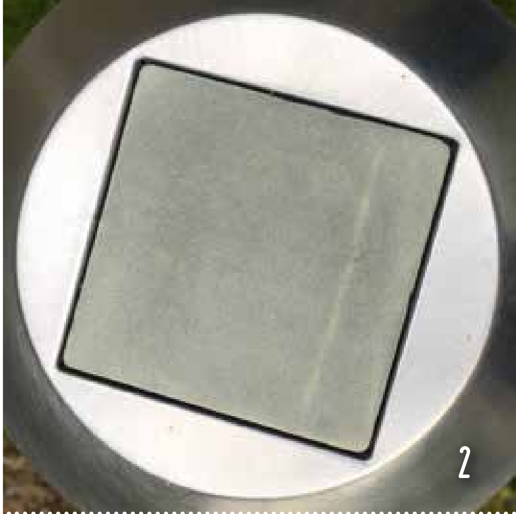
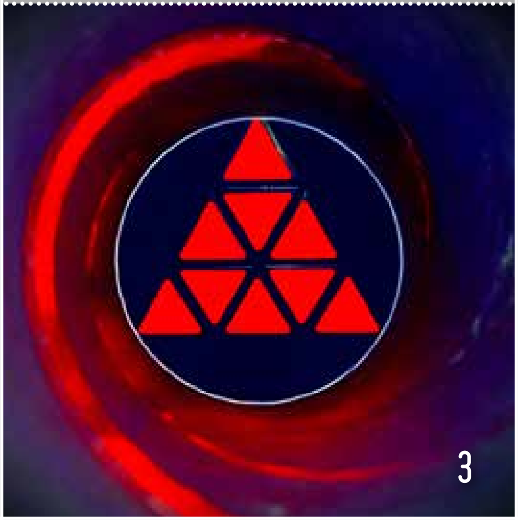
[22] G. Fauconnier, M. Turner, *Jak myślimy. Mieszaniny pojęciowe i ukryta złożoność umysłu*, translated by I. Michalska, Kronos, Warsaw 2019, p. 17.

[23] The original work is in Gallerie dell'Accademia Venezia, www.gallerieaccademia.it/en/node/1582.







Photography is the scholar's retina.

Jules Jansson

Mathematics begins when the child starts to distinguish between a triangle (conceived) and a triangle-shaped object.

Wacław Zawadowski

9 PROJECTIONS

Jean Piaget, criticising the common way of teaching, which consists in moving from the abstract to the concrete, postulates that the teaching of mathematics should be *prepared starting from kindergarten, through a series of manipulations, relating to sets, to numbers, to the concept of length and surface area*.[24] In the photograph (photo 2) we see a red circle and a grey triangle. But after all, *photography takes on meaning through the way it is used*.[25] The didactic sense of the installation is linked to its arrangement and exposition, which is to make the cone, the projection of which we see below the circle, invisible to the observer. The frames from experiments with light presented in the photographs encourage the formulation of problems concerning the transformation of parallelism into a focusing beam (photo 1), the conditions for obtaining an acute angle in the projection of a chilli pepper on a plane (photo 4), the difference between the visible (obtuse angle) and the known (right angle) (photo 5), or the scale of similarity during a homothetic enlargement of chess figures (photo 6).


1


3

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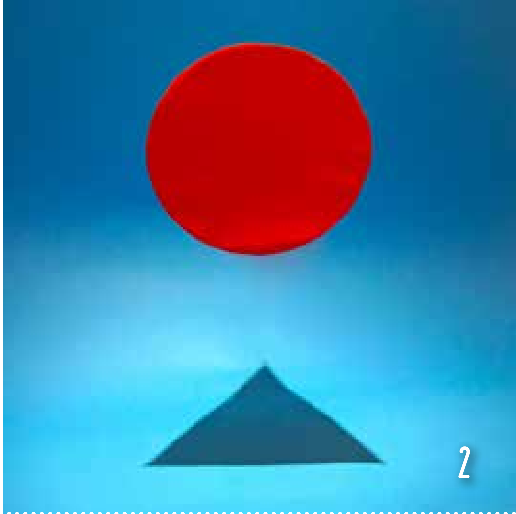
24 J. Piaget, *Dokąd zmierza edukacja*, translated by M. Domańska, PWN, Warsaw 1977, p. 87.

25 T. Barret, *Krytyka fotografii…*, p. 126.

If our mind manages to create a thing over which you have no greater, this thing will certainly exist.

Umberto Eco



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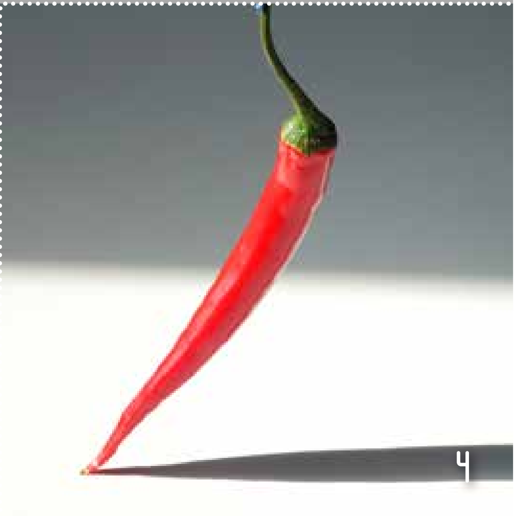


4

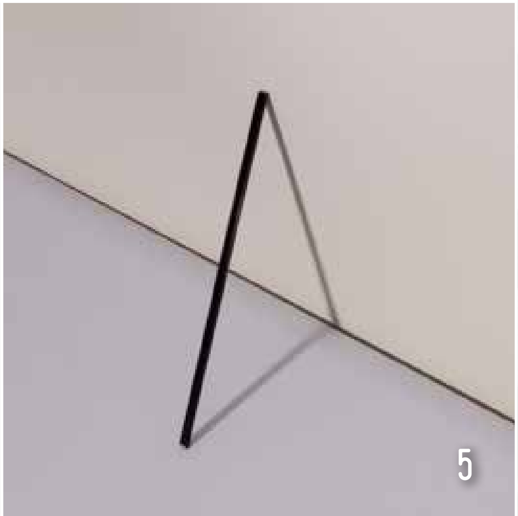
I have to create a reflection of what I felt when I looked at an object – not copy it.

Georgia O'Keefe

6

Mathematics is more of an activity than a science.

Luitzen Egbertus Jan Brouwer



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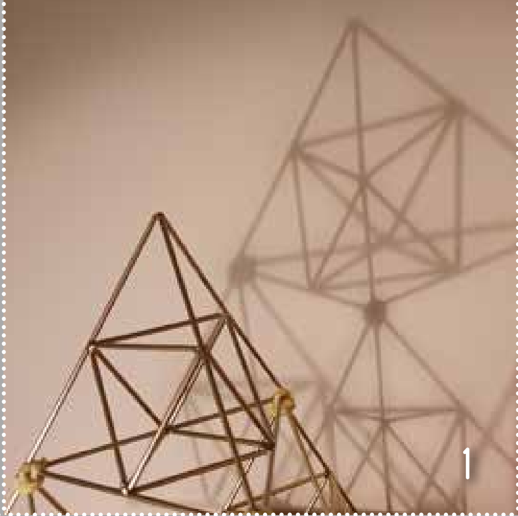
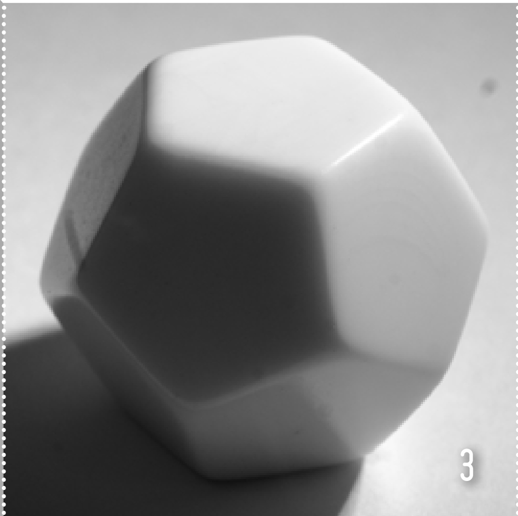
10

PLATONIC SOLIDS



Piotr Francuz's research has shown that the ways of viewing images by the so-called *novices* and the so-called *experts* differ fundamentally. The analyses of eye movements (saccades) when viewing still images showed that *what novices pay attention to has long ceased to interest experts*. […] *There follows a conclusion for those who create visual messages. If they target non-experts, they must take into account the effects of the bottom-up rather than top-down processing and interpretation of visual data by this audience.* […] *It is therefore most sensible to create conditions for the reception of visual messages that are not disturbed by uncontrolled lighting, distance or colour context*.[26] That is why photographs of platonic polyhedra are free of disturbing elements. The most important means of expression are lighting and framing. There are five regular polyhedra, as the Pythagoreans already proved: tetrahedron, cube (photo 3), octahedron, icosahedron (photo 5), and dodecahedron (photo 3, photo 4). *All the photographs have some meaning – without understanding what they connote, imply or suggest the viewer will not go beyond the obvious and will perceive them as reality, not as an image of reality.*[27] This is why the tetrahedron and octahedron are presented in one shot (photo 1) as an invitation for the reader to their own reading of the image of the ribbed model and its shadow.



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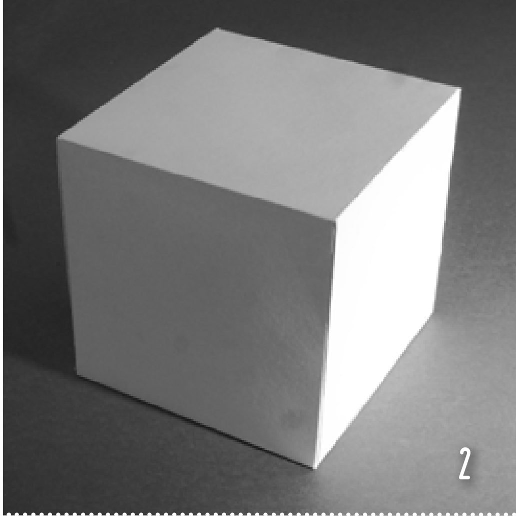
[26] P. Francuz, *Neuropoznawcze podstawy komunikacji wizualnej*, in: P. Francuz (ed.), *Komunikacja wizualna…*, pp. 44–45.

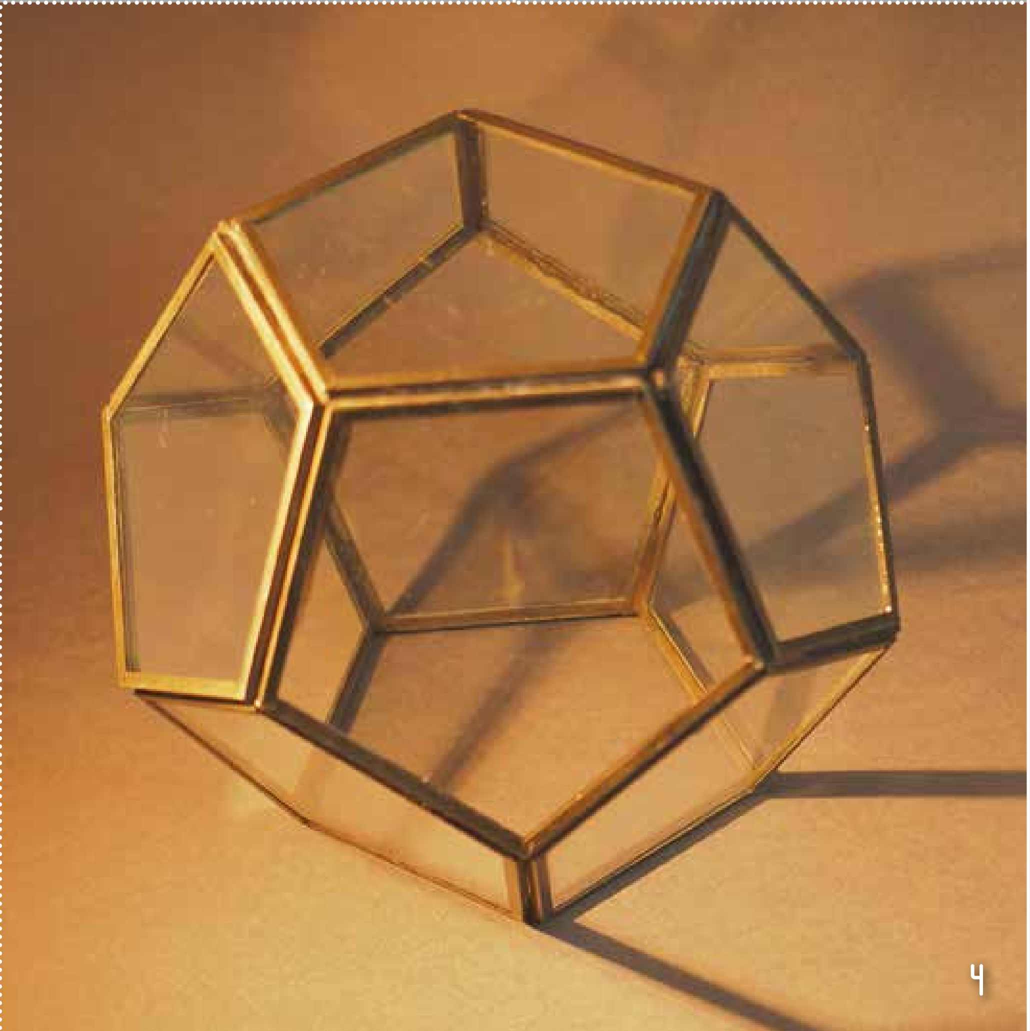
[27] T. Barret, *Krytyka fotografii…*, pp. 56–57.

Just as God breathes life into the cosmos through the 'fifth being' and into the four earthly elements […], so our Divine Proportion breathes life into the dodecahedron.

Luca Pacioli



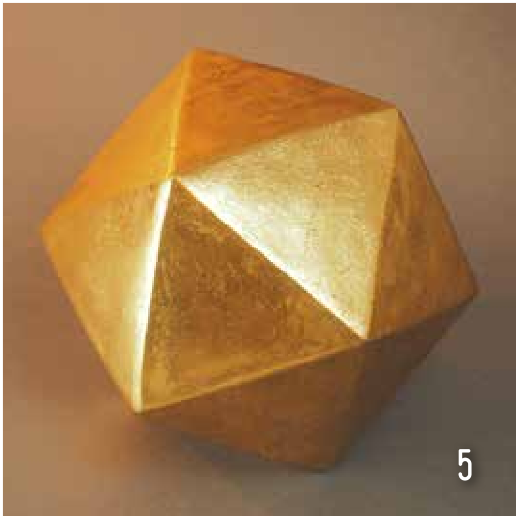
2



4

Life is like a keyhole. It is a single, tiny hole, through which one enters infinity.

Yann Martel



5

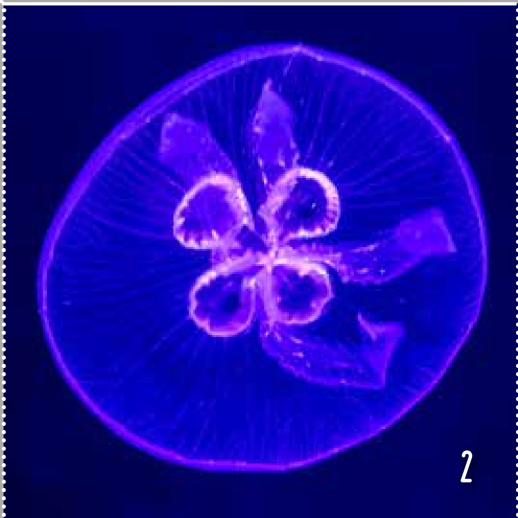
11 INFINITY

1

Mathematics is the science of infinity.

Hermann Weyl

The idea of actual infinity in mathematics is metaphorical in the sense that various cases of actual infinity utilise the metaphorical notion of the final result of a process that has no end.[28] Otton Nikodym drew attention to the processual nature of the use of the mathematical concept: *One understands (in one's own way) a given thing several times and each time this understanding is dispelled under the pressure of critical thought, which makes the mind capable of ascending to a higher cognitive level.*[29] Photography supports the creation and enrichment of cognitive structures. It encourages *asking questions and answering them, i.e. interpreting them.*[30] The photograph of a Möbius strip (photo 1) or an image of a tunnel of light (photo 4) provoke questions about the pursuit of infinity, while the photograph of a jellyfish (photo 2) graphically captures the sum of two infinite quantities, which is also an infinite quantity. The constellation (photo 3) refers to the Archimedean understanding of infinity as the number of grains of sand in the world. In relation to photography let us ask the following questions: what concepts, regularities and relationships can it bring us closer to? How far does the visualisation depart from the mathematical abstract? Is it possible to modify the shot, change it completely or even present a different idea to better convey the sense of a mathematical definition, theorem or relation by means of a picture?



2

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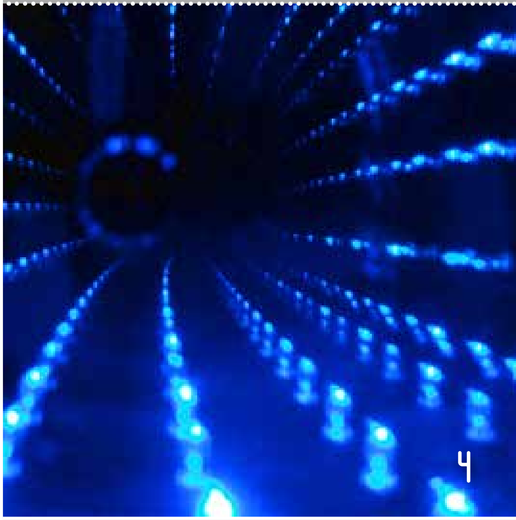
28 G. Lakoff, R. Núnez, *Where Mathematics Comes From*. Basic Books, New York 2000, p. 158.

29 O. Nikodym, *Dydaktyka matematyki czystej*, Lwów 1930.

30 T. Barret, *Krytyka fotografii…*, p. 53.

Mathematics: catching infinity red-handed.

Stefan Napierski



12 METAPHORS

Metaphors *trigger relevant imagery, allowing for better illumination and understanding of certain phenomena*.[31] Selected and presented metaphors due to combining an image with an apt verbal code make it possible to raise the level of reasoning. Researchers appreciate *the embodiment of mathematical concepts through the construction of cognitive representations based on ideas of dual coding*.[32] Their application in education, like the theory of synectics, allows *issues and problems to be approached from different points of view, to search for distant associations and connections*.[33] The photograph *The Integration of Tomato Soup* (photo 3) meets the conditions of the synectic method described by Wiesława Limont[34]. *Hausdorff's Autumn Space* (photo 1) perfectly illustrates the separate surroundings of points, and the *Middle of a Section*, not visible in photo 4, encourages considerations about the centre of gravity or the perpendicular bisector. The image depicting cake moulds (photo 2) – helps to understand the meaning of Dirichlet's pigeonhole principle.

To interpret is to give voice to signs that do not speak for themselves.

Hans-Georg Gadamer

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[31] W. Limont, *Poznawcze funkcje metafory*, in: M. Dudzikowa (ed.), *Filozofia pedagogice. Pedagogika filozofii*, "Colloquia Communia" 2003 (special issue), July–December, p. 168.

[32] M. Hohol, *Matematyka w metaforach? O wyjaśnianiu pojęć matematycznych za pomocą metafor kognitywnych*, in: M. Hetmański, A. Zykubek (ed.), *Metafory ucieleśnione*, Wydawnictwo Academicon, Lublin 2021, pp. 49–72 (https://doi.org/10.52097/acapress.9788362475810).

[33] W. Limont, *Uczeń zdolny. Jak go rozpoznać i jak z nim pracować*, GWP, Sopot 2010, p. 188.

[34] W. Limont, *Synektyka a zdolności twórcze, Eksperymentalne badania stymulowania rozwoju zdolności twórczych z wykorzystaniem aktywności plastycznej*, WN UMK, Toruń 1994, p. 16.

We must believe in the existence of the object depicted in the photograph, actually present in time and space.

André Bazin

MATH & art

CONCLUSION

We live in a world of media. Surrounded by images, which are supposed to replace words, accelerate our perception, impose the way of reasoning, feeling. *The uniqueness of mathematics is, among others, that its objects cannot be felt with the senses. They are invisible, they have no colour, no mass, no smell, no temperature.*[35] The use of photography in education brings mathematical abstraction closer to life. As Rudolf Arnheim writes, *even in the age of photography the machine did not replace imagination, but imagination put the machine into its service.*[36] A photograph, together with its description, treated as a pictorial-verbal metaphor, makes it possible to support the formation of mathematical knowledge on a symbolic level. It shortens the distance between formal objects and their visual representations, supports the development of students' mathematical imagination,[37] and *conveys meaning and allows for immediate understanding of complex phenomena in different symbolic systems.*[38]

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35 M. Makiewicz, *Opinia w przedmiocie diagnozy problemów związanych z procesem nauczania matematyki w szkole (począwszy od IV klasy szkoły podstawowej) wraz z odpowiednimi rekomendacjami*, in: *NIK o nauczaniu matematyki w szkołach* (https://www.nik.gov.pl/plik/id,19330,vp,21938.pdf, accessed on: 19.02.2019).

36 R. Arnheim, *Sztuka i percepcja wzrokowa. Psychologia twórczego oka*, translated by J. Mach, Słowo/obraz terytoria, Gdańsk 2004, p. 321.

37 M. Makiewicz, *Fotografia a wyobraźnia w kształceniu matematycznym. Raport z badań.* "Psychologia Wychowawcza" 51 (9), 2016, pp. 191–206.

38 W. Limont, *Wstęp*, in: W. Limont, B. Didkowska, *Edukacja artystyczna a metafora*, Wydawnictwo Naukowe Uniwersytetu Mikołaja Kopernika, Toruń 2008, p. 7.

1. HYPERCUBE

1. Maciej Pazera
2. Michalina Wysocka
3. Małgorzata Makiewicz
4. Adrian Pelic
5. Klaudiusz Łagodziński

5. SPIRAL

1. Aleksandra Szafrańska
2. Małgorzata Makiewicz
3. Monika Kucharska
4. Daniel Wójcik

9. PROJECTIONS

1. Donatella Baronchelli
2. Agnieszka Ławrynowicz
3. Anna Wróblewska
4. Aneta Woś
5. Emilia Turek
6. Borys Kosior

2. INFLECTION POINT

1. Wojciech Karwacki
2. Katarzyna Grządka
3. Giustyna Czosnyka
4. Łukasz Ungier
5. Tomasz Sczansny

3. FIBONACCI

1. Maciej Czapla
2. Angelika Leniak
3. Ignacy Różański
4. Tomasz Gałązka

4. FRACTAL

1. Krzysztof Karpiński
2. Michał Witkowski
3. Kacper Środoń
4. Maria Fiałkowska

6. PARALLELISM

1. Oliwia Sobiesiak
2. Giustyna Czosnyka
3. Anna Niemiec
4. Marek Miłoszewski

7. ADJACENT ANGLES

1. Joanna Superson
2. Norbert Sieczkiewicz
3. Mariusz Mączyński
4. Damian Kot
5. Marek Miłoszewski

8. INSCRIBED IN A CIRCLE

1. Hanna Laskowska
2. Wojtek Jablonski
3. Natalia Woźniak
4. Hanna Skupień

10. PLATONIC SOLIDS

1. Małgorzata Makiewicz
2. Małgorzata Makiewicz
3. Małgorzata Makiewicz
4. Małgorzata Makiewicz
5. Małgorzata Makiewicz

11. INFINITY

1. Jarosław Marcinek
2. Jolanta Szymczak
3. Łukasz Barzowski
4. Alicja Fertała

12. METAPHORS

1. Piotr Górny
2. Mateusz Motyl
3. Matusz Świerszcz
4. Ewa Woźniak